Chapter 3

Match factor extensions

For the mining industry, the match factor ratio is an important indicator with a dual purpose: during the equipment selection phase, it can be used to determine an appropriate fleet size such that the truck fleet productivity matches that of the loader fleet; when a fleet is selected, the match factor can be used to estimate the relative efficiency of the fleet. Thus far this ratio has been restricted to homogeneous fleets - however, heterogeneous fleets are common in large scale mines. We present several extensions to the match factor ratio to allow consideration of heterogeneous fleets. The results of this chapter have been published in the International Journal of Mining, Reclamation and Environment (Burt & Caccetta 2007).

3.1 Introduction

The mining and construction industries have long held interests in determining the productivity or efficiency of a selected fleet of trucks and loaders. One way to study the efficiency of a fleet is to weigh the efficiencies of the truck fleet and loader fleet against one another. The *match factor* is the ratio of truck arrival times to loader service rates.

The aforementioned industries have used the match factor for many decades as an indicator of productivity performance. As defined by Morgan & Peterson (1968), the match factor ratio, $MF_{i,i'}$, for trucks of type i working with loaders of type i' is given as

$$MF_{i,i'} = \frac{t_{i,i'} x_i}{\bar{t}_X x_{i'}} \ , \tag{3.1}$$

where x_i is the number of trucks of type i, $x_{i'}$ is the number of loaders of type i', $t_{i,i'}$ is the time taken to load truck type i with loader type i', and $\bar{t}_X$ is the average cycle time for all trucks. We provide a summary of the notation in section 3.6.

This ratio has hitherto relied on the assumption that the truck and loader fleets are homogeneous. That is, all the trucks are of the same type, and all the loaders are of the same type. In reality, mixed fleets are common. Heterogeneous fleets can occur when equipment types are discontinued, equipment is superseded, or simply when a mixed fleet is cheaper than a homogeneous fleet. Heterogeneous fleets can occur when new equipment is purchased to work alongside pre-existing equipment. It is also possible that a heterogeneous fleet can represent a minimum cost equipment selection solution. In this chapter we propose new ways for defining match factor for heterogeneous fleets.

In particular, we:

- Present two ways to define match factor when heterogeneous truck fleets are present [Section 3.2];
- Present a new method to define match factor when heterogeneous loading fleets are operating [Section 3.3]; and
- Present a new method to define match factor when both truck and loader fleets are heterogeneous [Section 3.4].

The aim of this chapter is to provide extensions to the productivity and efficiency measures currently available in the literature. This will enable greater consideration of heterogeneous fleets.

3.2 Heterogeneous truck fleets

The fleet most likely to be heterogeneous is the trucking fleet. This is due to the large number of trucks required to meet production requirements, compared to a relatively small number of loaders. Also, although there may often be different types of loaders operating in a mine, they are often in different locations and so can't be considered as separate fleets.

We begin by considering the truck arrival rate in the case of a heterogeneous truck fleet with a homogeneous loading fleet.

Definition 3.2.1 *The* ***truck arrival rate****, A, for a heterogeneous truck fleet with homogeneous loading fleet is the ratio of the number of trucks to the truck cycle time:*

$$A = \frac{\sum_i x_i}{\bar{t}_X} \ ,$$

where x_i is the number of trucks of type $i \in \mathbf{X}$ (where $\mathbf{X}$ is the set of all truck types), and $\bar{t}_X$ is the average cycle time for all truck types.

At this stage this rate is unaffected by the number of truck types, as we use an average truck cycle time. The loader service rate is the number of trucks that are served per second. The loader cycle time may vary between different truck types.

Definition 3.2.2 *The **loader service rate**, $D_{i'}$, for loader type i' is given by*

$$D_{i'} = \frac{x_{i'} \sum_i x_i}{\sum_i t_{i,i'} x_i} \ ,$$

where x_i is the number of trucks of type i, $x_{i'}$ is the number of loaders of type $i' \in \mathbf{X'}$, and $t_{i,i'}$ is the time required to load truck type i from loader type i'.

As the match factor is the ratio of truck arrival rate to loader service time, the match factor for heterogeneous truck fleets is easily derived from Definitions 3.2.1 and 3.2.2:

$$MF_{i'} = \frac{A}{D_{i'}} \tag{3.2}$$

$$= \frac{\sum_i x_i}{\bar{t}_X} \bigg/ \frac{\sum_i x_i x_{i'}}{\sum_i t_{i,i'} x_i} \tag{3.3}$$

$$= \frac{\sum_i t_{i,i'} x_i}{\bar{t}_X x_{i'}} \tag{3.4}$$

$$= \sum_i MF_{i,i'}. \tag{3.5}$$

It is clear that if only one truck type is operating in the fleet, then equation (3.4) will produce the same result as equation (3.1). Another way to think of this match factor for heterogeneous truck fleets is to add the individual match factors from each of the homogeneous sub-fleets. Note that the alternative method is only appropriate for the case of homogeneous loader fleets working with heterogeneous trucking fleets.

Sometimes we would like to use unique truck cycle times for different truck types in the fleet. This can occur when trucks have different routes. For example, consider a case where larger equipment is used to haul waste while smaller trucks are used to haul ore: the waste and ore may be sent to different locations, with significantly different cycle lengths. When individual truck cycle times are used, the times must be weight averaged to produce an accurate match factor. Equation (3.4) can be easily extended to account for unique truck cycle times.

Definition 3.2.3 *The **average cycle time**, $\bar{t}_X$, is given by:*

$$\bar{t}_X = \frac{\sum_i t_i x_i}{\sum_i x_i} \ ,$$

where t_i is the cycle time for truck type $i \in \mathbf{X}$ and x_i is the number of trucks of type i.

Now, substituting this new truck cycle time into equation (3.4), we have the following lemma:

Lemma 3.2.1 *For heterogeneous truck fleets with individual truck cycle times, the **match factor** for homogeneous loader fleets of type $i' \in \mathbf{X}'$ can be represented by*

$$MF_{i'} = \frac{\left(\sum_i x_i\right)\left(\sum_i t_{i,i'} x_i\right)}{x_{i'} \sum_i t_i x_i} \ . \tag{3.6}$$

3.3 Heterogeneous loader fleets

This section considers the case of mixed loaders in the fleet, while the trucks remain uniform in type. The time required to load a truck may be different for various types of loaders. The loader service rate is the number of trucks served in a defined time period. In a heterogeneous fleet, the time taken to serve a truck may differ between the varying loader types.

Lemma 3.3.1 *The **loader service rate** for heterogeneous loader fleets working with truck type $i \in \mathbf{X}$ is given by*

$$D_i = \sum_{i'} \frac{x_{i'}}{t_{i,i'}} \ . \tag{3.7}$$

Proof: The loader service rate is the ratio of the total number of trucks to the time required to serve them. We have $t_{i,i'}$ for several loader types i' and one truck type i. The number of trucks type i served by loader type i' in $t_{i,i'}$ time is:

$$\frac{1}{t_{i,i'}} \ .$$

Thus the total number of trucks served by all loader types in a unit of time is

$$D_i = \sum_{i'} \frac{x_{i'}}{t_{i,i'}} \ , \text{ as required.}$$

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Recall that the match factor is the ratio of truck arrival rate to loader service rate. The truck arrival rate is:

$$A_i = \frac{x_i}{t_i} \ ,$$

which gives the following theorem:

Theorem 3.3.2 *For heterogeneous loader fleets, the* ***match factor*** *for a homogeneous truck fleet of type $i \in \mathbf{X}$ is*

$$MF_i = \frac{x_i}{t_i \sum_{i'} \frac{x_{i'}}{t_{i,i'}}} \ . \tag{3.8}$$

When only one type of loader operates in the fleet, equation (3.8) reduces to equation (3.1). In the case of multiple dump locations or routes, equation (3.8) can be expanded to account for differing truck cycle times. First, we represent the average truck cycle time by:

$$\bar{t}_X = \frac{\sum_h t_{i,h} x_{i,h}}{x_i} \tag{3.9}$$

where $t_{i,h}$ is the cycle time for truck type i on route h, and $x_{i,h}$ is the number of trucks of type i working on route h. This gives the following corollary:

Corollary 3.3.3 *The* ***match factor*** *for heterogeneous loader fleets working with truck type i (with individual truck cycle times for trucks on route h) can be represented by*

$$MF_i = \frac{(x_i)^2}{\left(\sum_{i'} \frac{x_{i'}}{t_{i,i'}}\right)\left(\sum_h t_{i,h} x_{i,h}\right)} \ . \tag{3.10}$$

3.3.1 Example

The following example calculates the match factor of a heterogeneous loader fleet. Table 3.1 outlines the equipment set.

	Equipment	Capacity (Tonnes)	Cycle Time (seconds)
22	Truck type A	150	1500
1	Loader type B	60	35
1	Loader type C	42	35

TABLE 3.1: Example data for a heterogeneous loader fleet with common truck cycle time.

The cycle time for the loader is the time taken for one full swing of the bucket. Some trucks may need several buckets to fill their trays. The first step is to determine the unique loading time for each truck. If the truck capacity is not a round multiple of the loader capacity, then we take another scoop if the capacity left is more than one third of the loader bucket size. This is because it takes almost the same amount of time to move a portion of a scoop as it does to move a full scoop (Gransberg 1996).

Truck type A and loader type B:	$\frac{150}{60} = 2.5$	3 swings	$3 \times 35 = 105$ seconds
Truck type A and loader type C:	$\frac{150}{42} = 3.6$	4 swings	$4 \times 35 = 140$ seconds

This gives the match factor:

$$\begin{aligned} MF_A &= \frac{x_A}{\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{A,i'}}} \\ &= \frac{22}{1500 \times (\frac{1}{105} + \frac{1}{140})} \\ &= 0.88. \end{aligned}$$

This shows that the fleet is under-trucked. When a minimum cost fleet is desired, one would reasonably expect that under-trucking would provide better solutions than perfectly matching the fleets with a match factor of 1.

3.4 Heterogeneous truck and loader fleets

For heterogeneous truck and loader fleets, we consider the time required for each loader to serve the available truck fleet. This is equal to the sum of the number of trucks of type i multiplied by the time required to serve that truck type. We call this the *loading times*, $t_{i'}$, for each loader type i'.

Definition 3.4.1 *The time, $t_{i'}$, required for a loader of type i' to serve the entire fleet of trucks is*

$$t_{i'} = \sum_i t_{i,i'} x_i \ . \tag{3.11}$$

So the time taken for one loader to serve one truck is:

$$\frac{t_{i'}}{\sum_i x_i}. \tag{3.12}$$

Thus we obtain the following corollary:

Corollary 3.4.1 *The **loader service rate** for heterogeneous trucks and loaders is* *given by*

$$D = \sum_{i'} \frac{x_{i'} \sum_i x_i}{t_{i'}} \ . \tag{3.13}$$

As in Section 3.3, the truck cycle time is assumed to be constant for the entire truck fleet for that period.

Theorem 3.4.2 *The* ***match factor*** *for both heterogeneous truck and loader fleets can be represented by*

$$MF = \left(\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{i'}} \right)^{-1} \ . \tag{3.14}$$

Proof: We now consider the truck arrival rate for the entire fleet, given in definition 3.2.1:

$$\begin{aligned} MF &= \frac{A}{D} \\ &= \frac{\sum_i x_i}{\bar{t}_X} \times \frac{1}{\sum_i x_i \sum_{i'} \frac{x_{i'}}{t_{i'}}} \\ &= \frac{1}{\bar{t}_X \sum_{i'} \frac{x_{i'}}{t_{i'}}} \ , \text{ as required.} \end{aligned}$$

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If we have unique truck cycle times, equation (3.14) can be easily extended to:

$$MF = \frac{1}{\left(\sum_{i'} \frac{x_{i'}}{t_{i'}} \right)} \sum_{i \in I} \frac{x_i}{(t_{i,I} x_{i,I})} \tag{3.15}$$

where $t_{i,I}$ is the unique truck cycle time for trucks in subset $I \in \mathbf{X}$.

We find an expression that is equivalent to equation (3.14) but may be simpler to implement in a spreadsheet by first observing that the loader service rate can be represented by:

$$D = \frac{\sum_{i'} x_{i'} \sum_i x_i \prod_{h \neq i'} (t_{i,h} x_i)}{\prod_{i,i'} (t_{i,h} x_i)} \ .$$

We take the truck arrival rate for the entire fleet from definition 3.2.1 to obtain the following theorem.

Theorem 3.4.3 *The match factor for both heterogeneous truck and loader fleets can be represented by*

$$MF = \frac{\prod_{i,i'} t_{i,i'} x_i}{\prod_{h \neq i'} (t_{i,h} x_i) \sum_{i'} x_{i'} \bar{t}_X} \ . \tag{3.16}$$

When only one type of truck and one type of loader operate in the fleet, equations (3.14)-(3.15) reduce to equation (3.1), as expected.

3.4.1 Example

This example determines the match factor of a heterogeneous truck and loader fleet. Table 3.2 presents the data set.

	Equipment	Capacity (Tonnes)	Cycle Time (seconds)
15	Truck type A	150	1500
7	Truck type B	230	1500
1	Loader type C	60	35
1	Loader type D	38	30

Table 3.2: Example data for a heterogeneous truck and loader fleet with common truck cycle time.

The unique loading times for each truck are determined by the rule of thumb described in Section 3.3.

Truck type A and loader type C: $\frac{150}{60} = 2.5$ 3 swings $3 \times 35 = 105$ seconds

Truck type A and loader type D: $\frac{150}{38} = 3.9$ 4 swings $4 \times 30 = 120$ seconds

Truck type B and loader type C: $\frac{230}{60} = 3.8$ 4 swings $4 \times 35 = 140$ seconds

Truck type B and loader type D: $\frac{230}{38} = 6.1$ 6 swings $6 \times 30 = 180$ seconds

We calculate the *loading times*, $t_{i'}$, for each loader of type i'.

$$t_C = 15 \times 105 + 7 \times 140 = 2555$$
$$t_D = 15 \times 120 + 7 \times 180 = 3060$$

$$\begin{aligned} MF &= \frac{1}{\left[\frac{1}{2555} + \frac{1}{3060}\right] \times 1500} \\ &= 0.928 \end{aligned}$$

This solution is close to the theoretical perfect match of 1.0. This is a good result in terms of overall efficiency and productivity of the fleet. However, one should be aware that costing has not been considered in determining the match factor and so it is possible that the fleet would be cheaper to operate even if the match factor was lower.

3.5 Discussion

For the mining industry, the match factor ratio is an important indicator which we have extended for several likely circumstances, including heterogeneous truck and loader fleets with multiple routes. The match factor can be used to optimise the truck cycle time in order to gain maximal efficiency from the selected fleet. Alternatively, project managers may use the match factor formula to determine the ideal number of trucks in the fleet. The formulae presented in this chapter are less restricted in their choice of equipment, select mixed fleets to suit the productivity requirements and minimise materials handling expense.

It is interesting to note that the Morgan & Peterson (1968) match factor ratio excludes waiting and queuing times for trucks and loaders. This may be because the waiting time for a truck fleet is difficult to estimate without first knowing the size of the truck fleet. However, if we use the match factor ratio as an index of overall fleet efficiency, then it is acceptable to include waiting times that have been estimated by other methods.

The formulae presented in this chapter provide a sensible extension to the original equation and bring greater accuracy to the cases where mixed fleets operate together. All of these formulae can be implemented easily in spreadsheet software such as Microsoft Excel. Throughout the rest of this thesis, we employ the heterogeneous match factor ratios to indicate the overall efficiency of the selected fleets.

3.6 Summary of notation

$\mathbf{X}$	the set of all available truck types.
$\mathbf{X}'$	the set of all available loader types.
i	the truck equipment type index, $i \in \mathbf{X}$.
i'	the loader equipment type index, $i' \in \mathbf{X}'$.
$MF_{i,i'}$	the match factor for homogeneous truck and loader fleets of types i and i' respectively.
$MF_{i'}$	the match factor for heterogeneous truck fleets working with loader type i'.
MF_i	the match factor for heterogeneous loader fleets working with a homogeneous truck fleet of type i.
MF	the match factor for heterogeneous truck and loader fleets.
x_i	the number of trucks of type i.
$x_{i'}$	the number of loaders of type i'.
t_i	the cycle time for truck type i.
$t_{i'}$	the time required for a loader of type i' to serve the fleet of trucks.
$t_{i,i'}$	the the time taken to load truck type i with loader type i'.
$\bar{t}_X$	the average cycle time for all the trucks.
A	the truck arrival rate.
A_i	the truck arrival rate for truck type i.
$D_{i'}$	the loader service rate for heterogeneous truck fleets working with loader type i'.
D_i	the loader service rate for heterogeneous loader fleets working with truck type i.
D	the loader service rate for heterogeneous loaders working with a heterogeneous truck fleet.