SCENARIO-SCAPING WITH INVERSE PROGRAMMING FOR INTERMODAL TRANSPORATION

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Abstract. We study an intermodal transportation problem over a long-term planning horizon. Due to uncertainty in many aspects of the problem, such as politics, future demand, interest rates and costs, we establish several possible scenarios. In a planning situation it is ideal to know when one scenario becomes preferable to another. In this paper we make use of inverse programming techniques to sketch the landscape of the uncertain scenario cost function to aid future planning.

1. Introduction

In this paper we study the problem of choosing between scenarios over a long-term planning horizon for an intermodal transportation problem. In particular we wish to determine the conditions under which one scenario becomes preferable to another when some parameters in the problem are uncertain.

A leading European construction group currently delivers a product from a handful of factories via a trucking network to several destinations. The company is studying a series of future expansion scenarios. There are opportunities to increase the number of factories supplying product and opportunities to expand the network to an intermodal transportation network by adding rail and/or barge links to the network. These are long-term planning commitments: the company wishes to understand the conditions under which each scenario becomes ideal.

Intermodal transportation problems involve combining two or more forms of transportation in the one network. Since the capacities of these different modes will, by design, be different, there will be consolidation points in the network. The arrival times of flexible services to consolidation points; the timing of large capacity services; and the regularity/frequency of large capacity services are all important and interesting considerations.

In this paper we propose a method for sketching the space of optimal conditions for all imagined scenarios for the capacitated network design problem. Since the cost vector may vary according to some unknown distribution, we wish to determine the conditions (of the cost vector) under which alternate scenarios become preferable. To do this, we begin with the current known costs. We iteratively select scenarios and use an inverse programming method to determine the cost vector that will optimise each given scenario. This information can assist future planners who have sensitive knowledge about the problem, such as contract negotiation processes—they can try to negotiate the cost conditions that make the scenario optimal as described by our solution.

Network planning problems are typically modelled as capacitated multi-commodity network flow problems, which are known to be difficult to solve due to weak linear programming relaxations [2]. Adding service, location and equipment selection into this problem only increases the dimension and exacerbates solving power—this is predominantly due to the introduction of transition constraints in the time step, which bring special-ordered-set constraints (type 1) to the formulation. But then this is also because the selection variables are all binary and also dimensioned in time steps—the sheer quantity of binary variables is inhibiting to any Simplex-based solver [3]. As our heuristic involves solving two programs per scenario, we focus on developing reduced models (including fixing to a linear program) to improve tractability and use approximate solutions where practicable. We refer the reader to [4] for a recent comprehensive survey on methods for capacitated network design problems.

2. Method

We define our transportation network on a graph $G = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ is the set of nodes representing possible origins and destinations of the service paths (including split nodes where interchange and holding may occur), and $\mathcal{A}$ is the set of arcs representing service paths (including holding paths) between the nodes. Let k be the proportion of (maximum sized) service used (including holding) provided by a particular type of equipment. Then the triplet (i, j, k) represents a service path in the network (from node i to j, with equipment type k). Notably, the arc represents each service provided and is capacitated by the maximum possible capacity of a single service—the proportion denoted in the solution thereby selects the number of trailers on a train or size of trailer on a truck, for example. We model the network design problem as a capacitated multi-commodity network flow problem, with flow variables creating a distribution schedule from factories to destinations such

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that demands at the destinations are satisfied. The model also includes binary equipment selection variables: flow and service variables are continuous or semi-continuous—flow and services can be prevented from splitting using SOS1 constraints.

In each scenario we will fix a large number of variables. Importantly for the efficacy of our proposed procedure, all integer variables must be fixed (and in a feasible way): the inverse programming method we employ is valid for linear programs. This technique is analogous to sensitivity analysis with the proviso that we start with a sub-optimal solution and seek changes in parameters that will make the solution optimal. It is based on the notion that the new cost vector only changes when the primal variables are non-zero—we know from complementary slackness that this only happens when the primal constraints are binding. This leads to a set of equations that provide the perturbed cost vector in terms of the dual variables.

Suppose that we have a set of scenarios that in some way fix the equipment and location in the model. For example, in one scenario we might introduce a rail link between two major origin-destination clusters and employ several new factories. Thus we know that the equipment types are limited to truck and rail; and the locations (nodes and arcs) are limited to the current locations plus the new considered locations. All other equipment and location (i.e. arc) variables will be zero. In our model-sketch, the equipment types will be a part of the scenario (i.e. this aspect of the problem may be solved heuristically *a priori*). The dependency of services and subsequent flow on the type of equipment means that related service and flow variables may also be fixed to zero. Thus we obtain a reduced version of the original problem.

We denote this reduced primal problem for each scenario by $\mathcal{P}^s$. Then, to find a flow solution (and a solution for remaining un-fixed service) for each scenario we need only solve $\mathcal{P}^s$ exactly or approximately to obtain x^0. We assume that x^0 is not the optimal solution for the current costing situation. We are therefore interested in determining the cost vector that would make the given scenario (and our calculated x^0) optimal. In other words, under what conditions should we choose that scenario? Furthermore, we wish to know the *nearest* solution for the objective function parameters that obtain this result. We use inverse programming (for the profit vector under the L_1 norm), as described in [1], to determine this perturbation in the cost vector.

Algorithm 1 Scenario-scaping with inverse programming.

1: Define the capacitated network design problem as a mixed-integer program, $\mathcal{P}$, with objective function z.
2: **for** each scenario $s \in \mathcal{S}$ **do**
3: Heuristically determine the equipment selection solution for s and denote this h_s.
4: Fix all equipment variables according to h_s—this fixes all integer variables in the model. Fix some of the continuous variables (flow, service path) as implied by h_s and the location solution in s.
5: Solve the remaining reduced LP problem $\mathcal{P}^s$ (exactly or approximately) with z to obtain the full scenario solution, x^0. Note that x^0 is now feasible for $\mathcal{P}$.
6: Consider the set of binding constraints, $\mathcal{B}$, for x^0 in $\mathcal{P}$. Remove all non-binding constraints to obtain the reduced problem, $\mathcal{P}^s_B$—keep all integer variables fixed according to h_s but restore the bounds on all continuous variables.
7: Solve the reduced problem $\mathcal{P}^s_B$ exactly. Note that, different to $\mathcal{P}^s$, $\mathcal{P}^s_B$ does not contain any fixed continuous variables.
8: Let the vector π be the optimal dual variables associated with the constraints in $\mathcal{B}$. The elements of the new cost vector for scenario s are given by:

$$d_j^* = \begin{cases} c_j - |c_j^\pi| & \forall \{j : c_j^\pi > 0 \text{ and } x_j^* > 0\} \\ c_j + |c_j^\pi| & \forall \{j : c_j^\pi < 0 \text{ and } x_j^* < u_{i,j}\} \\ c_j & \text{otherwise} \end{cases}$$

where $c_j^\pi = c_j - \sum_{i \in \mathcal{B}} a_{i,j}\pi_i$; $u_{i,j}$ is the upper bound on variable j in $\mathcal{P}$; and i corresponds to constraints in $\mathcal{P}$.
9: **end for**

We observe, as in [1], that if all the constraints in $\mathcal{P}$ are binding at step (6), then the dual inverse problem is identical to the original problem. We note that the amount of 'fixing' that occurs in the procedure depends entirely on how different the stated scenario is from the current scenario. We also note that solving the reduced formulations at steps (5) and (7) is non-trivial. However, only step (7) must be solved to optimality for the inverse programming step to be valid.

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