

# MUMS Problem Solving Competition

Melbourne University Mathematics and  
Statistics Society

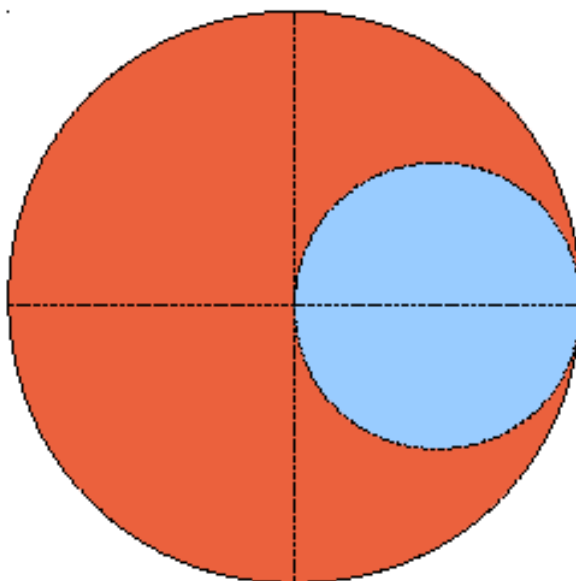
27 June, 2012

## RULES

- Teams must have either three or four competitors.
- The competition will consist of five rounds, each lasting ten minutes.
- In each round there are five questions, but only your *best three* questions will contribute to your score. The questions have 3, 4, 5, 6 and 7 points allocated respectively.
- You will not lose points for incorrect answers.
- Prizes will be awarded to the top three school teams.
- You must have fun!

## ROUND ONE

1. What is the highest possible score in this competition?
2. How many ways are there to arrange the letters of "SIX" in a line?
3. Assuming the radius of the larger circle is 6, what is the ratio of the area of the smaller circle to the bigger circle?



4. Thara is having a house party. By 10:00pm, half of the guests leave. At 11:00pm, a third of the remaining guests leave. At midnight, a quarter of the remaining guests also leave. If 9 guests are still at the party, how many guests did Thara have initially?
5. Each of the numbers in the set  $\{1, 2, 3, 4, 5, 6\}$  can be put into boxes exactly once to make the following equation correct. What is the number on the right hand side of the equation?

$$\square\square \times \square = \square\square\square$$

## ROUND TWO

1. What is the largest odd factor of 2012?
2. David and Jeff share a car and always drive to the shops. In one month, Jeff went to the shops 5 times, 3 of which he went by himself. If David went to the shops 12 times in that month, the number of times he was the driver is one of three possible numbers. What are those numbers?
3. MUMS has 5 executive committee members who are at least 15 years old. Twelve years ago, the product of their ages was 29645. What is the sum of their ages now?

4. Dan has four flowerpots of colour; red, green, blue, yellow. He also has one flower of each of those colours. How many different ways can he arrange the flowers into flowerpots without having the flower colour matching the flowerpot colour?

Note: Exactly one flower must be in each pot.

5. Two different right angled triangles with integer sidelengths have the property that their areas are equal to their perimeters. What are the sidelengths of these two triangles?

## ROUND THREE

1. How many numbers between 1 and 2012 are divisible by 5?
2. Dougal is lost in a forest and needs to get back to his campsite. His campsite is exactly two kilometres north and two kilometres east from his current position. Unfortunately Dougal walks three kilometres west. How far away from his campsite is he now?
3. Sam is quite a quick runner. He runs at 6 metres per second. Mel can only manage to run at 4 metres per second. Given that Mel starts 150 metres ahead of Sam, and they both start running at the same time, how far has Sam run before he catches Mel?

4. In the town of Richard Berry, there are 3 policemen and 10 thieves. Every 3 minutes on the dot, each policeman captures a thief if there is one free to catch. Every four minutes on the dot, exactly one thief escapes from jail. If one thief starts off in jail, how long will it take until all the thieves are in jail for the first time?
5. How many ways are there to make the word ALGEBRA, going from top to bottom?

A  
L L  
G G G  
E E E E  
B B B  
R R  
A



## ROUND FOUR

1. Simplify

$$\frac{6\sqrt{6}}{\sqrt{6} - \frac{1}{\sqrt{6}}} - 6$$

2. If I have an upright triangle, and I make a horizontal cut exactly halfway up the triangle, what is the ratio of the top area compared to the bottom area?
3. Given a cube with volume  $216\text{cm}^3$ , what is the furthest distance any two points on the cube can be from one another?
4. Michael is thinking of three numbers whose sum is 27. When he adds just the first and second numbers he gets 16. When he adds the second and third numbers he gets 20. What are the three numbers that Michael is thinking of?

5. Franky and Andrew play a game with two dice. In this game, Franky only knows what the sum of the dice are, while Andrew only knows what the product of the dice are. Andrew and Franky then take turns saying “I don’t know what the roll was” or “I know what the roll was” - they can’t lie. The game ends when one player knows what numbers were rolled. If Franky starts off, after how many turns in total is the game guaranteed to end?

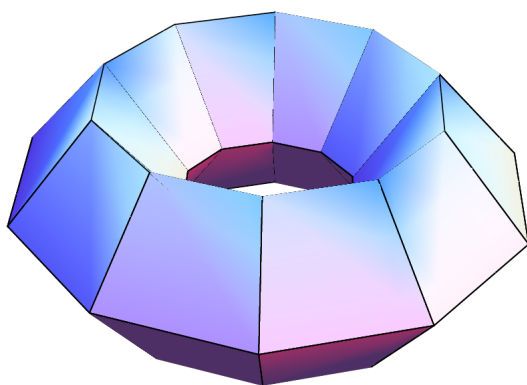
## ROUND FIVE

1. Aaron folds a piece of paper 10 times, making sure to have the piece of paper lie flat on the table after every fold. If Aaron then unfolds his paper and colours in the folded regions so that no two adjacent regions have the same colour, what is the minimum number of colours he will need?
2. Han baked himself a pizza. Since he has weak arms, he can only cut it 5 times. What is the maximum number of pieces of pizza that he can make with straight cuts?
3. The sum of two numbers is 6 and the product of these two numbers is 6. What is the sum of the squares of their reciprocals?

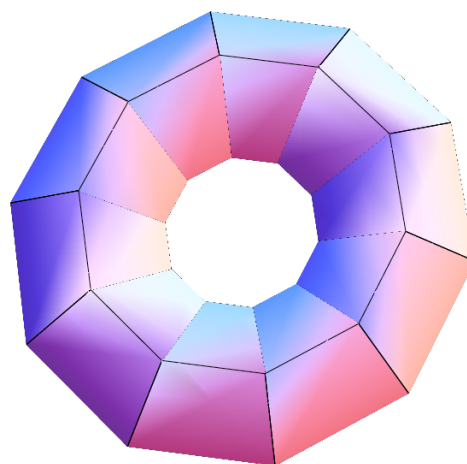
4. The Euler Characteristic of a surface is

$$V - E + F$$

where  $V$  is the number of vertices,  $E$  is the number of edges and  $F$  is the number of faces. What is the Euler Characteristic of the surface below?



Side view



Top view

5. There is only one 3 digit number which is equal to 15 times the sum of its digits. What is it?

## TIE-BREAKER

1. Two teams  $A$  and  $B$  are in a tiebreaker. If the question is from (1) geometry, (2) algebra, or (3) trigonometry, then team  $A$  wins. If the question is from (4) combinatorics or (5) calculus then team  $B$  wins. If the topic is from (6) probability, then neither team wins because probability is too hard and another question is asked.

If a fair die is rolled to determine the topic, what is the probability that team  $A$  wins?