MIT Challenge 2014 Solutions

Task 1.1

All vertices are of degree 4, except for the three vertices at the corners of the biggest triangle, which are of degree 2.

Task 1.2

a) 0   b) 0   c) 2   d) 2   e) 4   f) 4

There are always an even number of odd vertices in any graph.

Solutions

Task 1.3

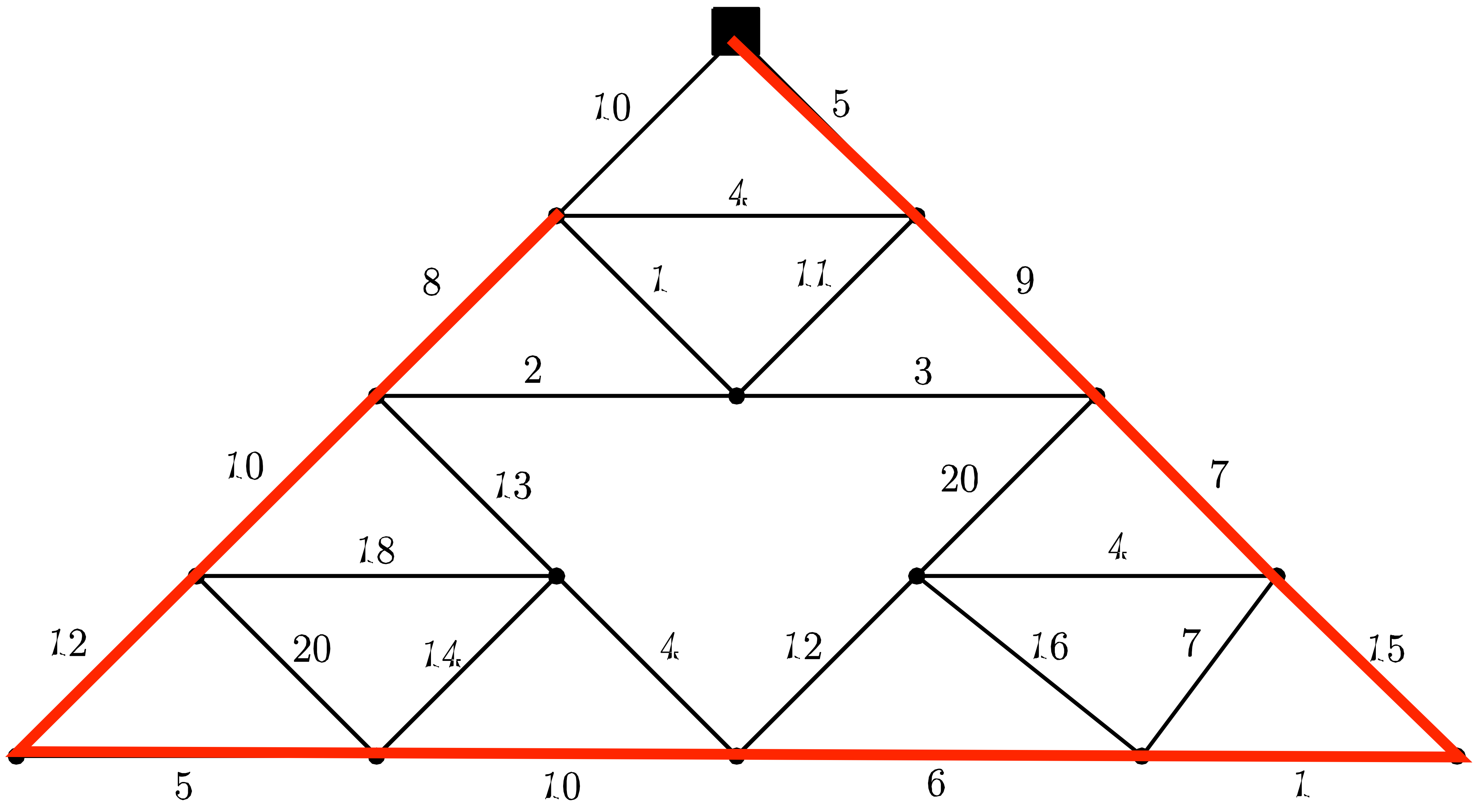
Suppose we add up the degrees of the vertices of a graph. Then each edge is counted twice, and therefore the sum is even. Therefore there must be an even number of odd vertices.

Task 1.4

There are many answers. Eg. 5, 9, 7, 15, 1, 6, 10, 5, 12, 10, 8, 4, 11, 2, 13, 14, 20, 18, 4, 12, 4, 7, 16, 20, 3, 1, 10

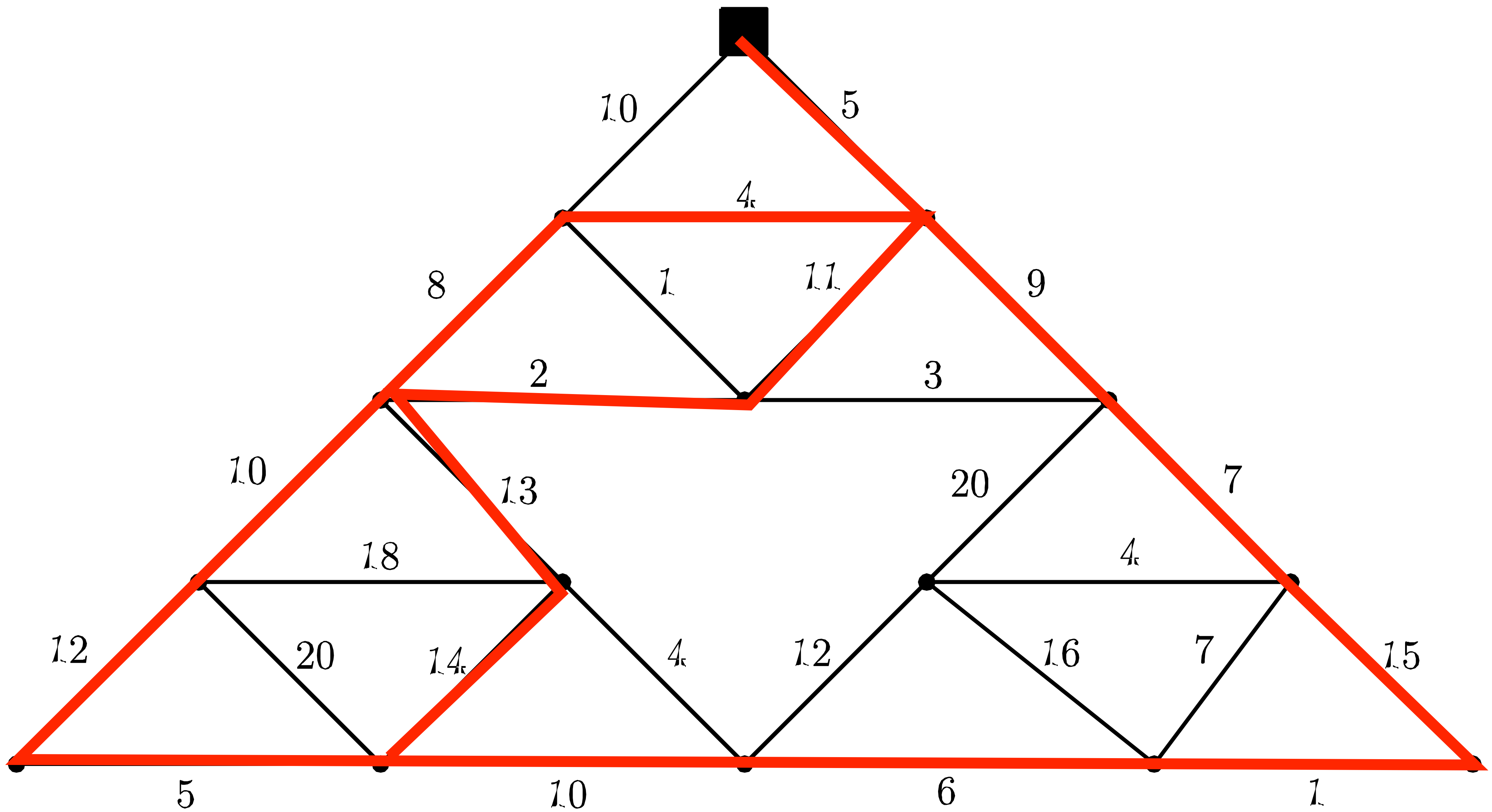
Solutions

Task 1.4



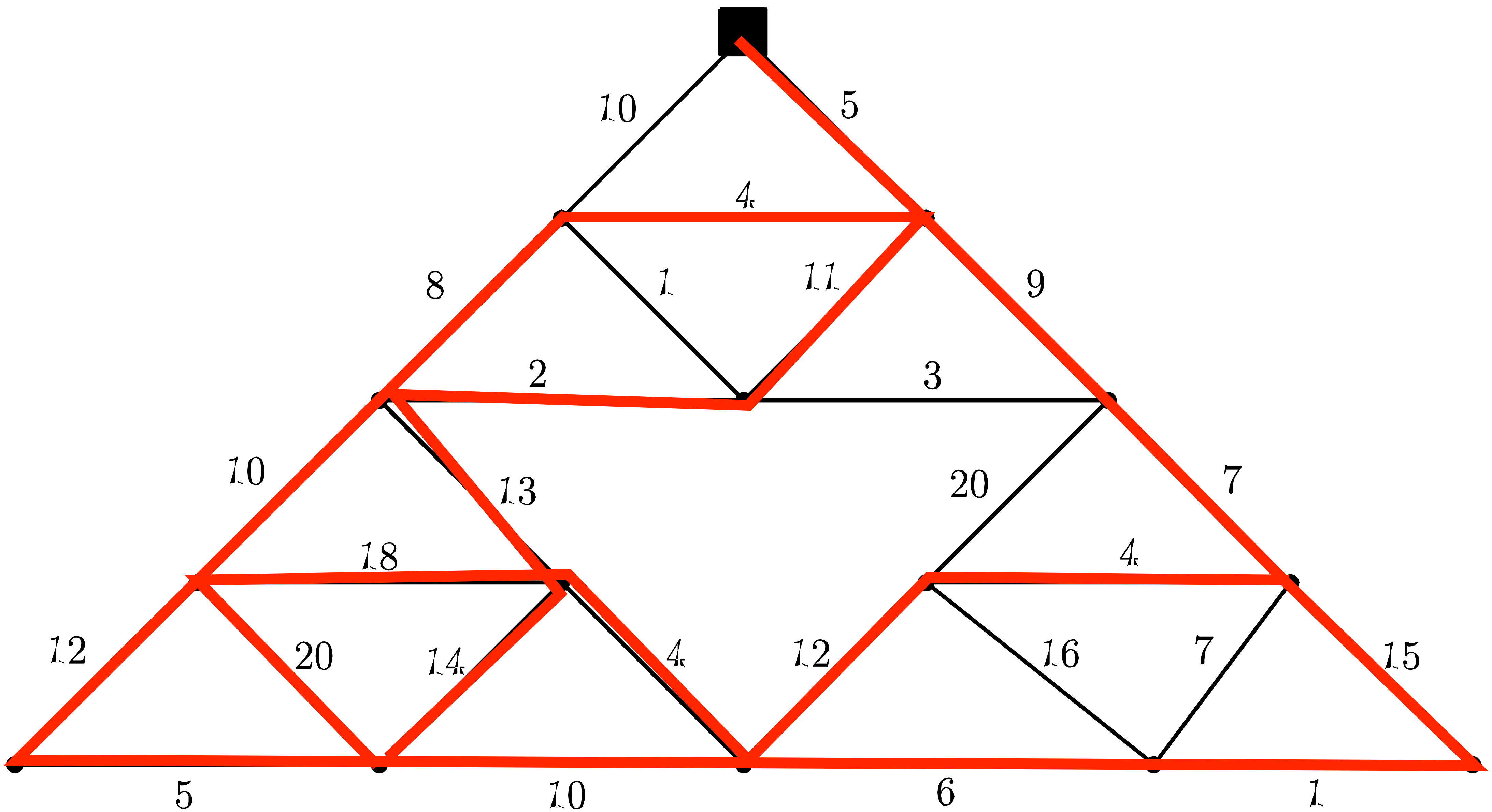
Solutions

Task 1.4



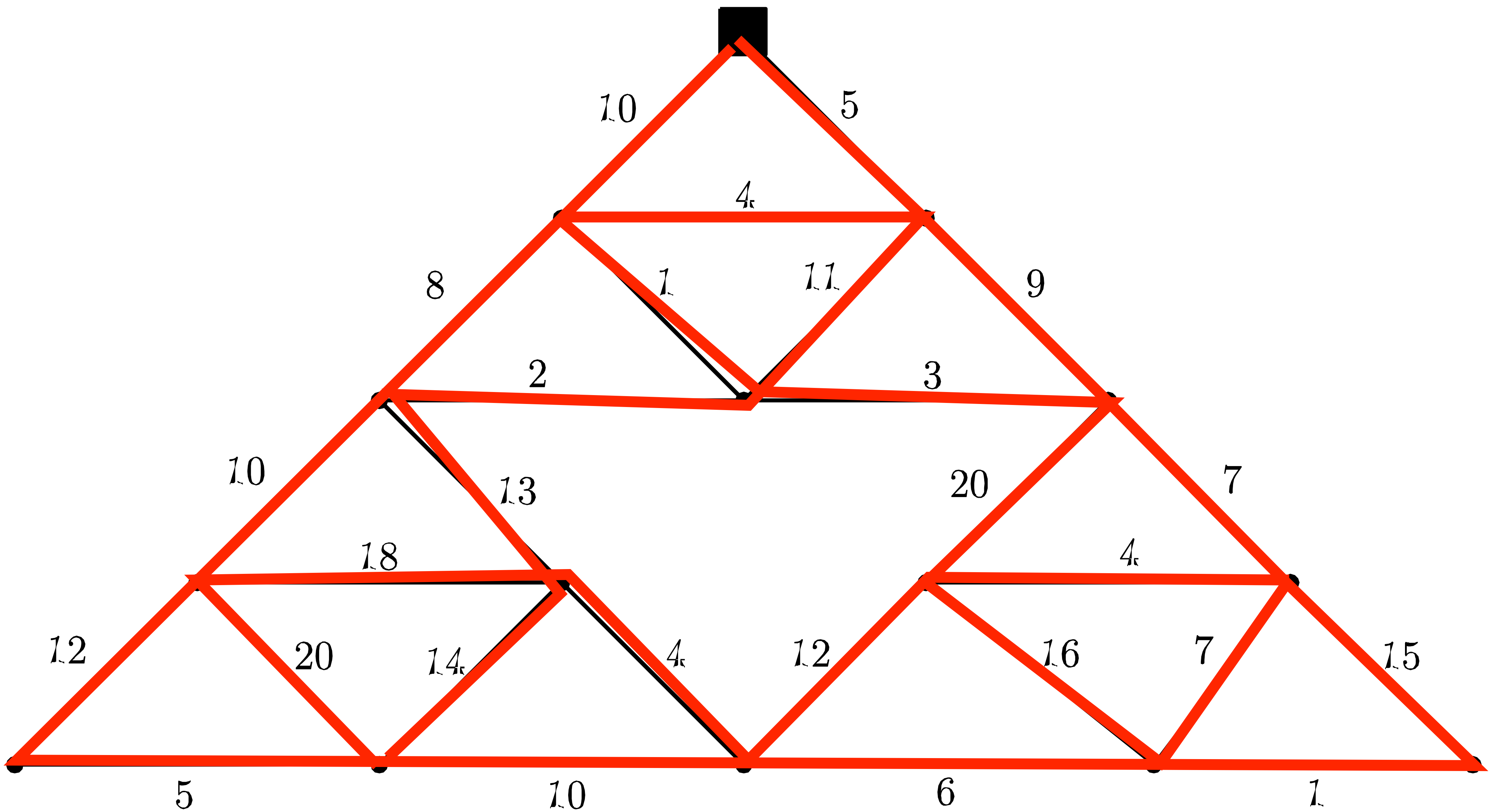
Solutions

Task 1.4



Solutions

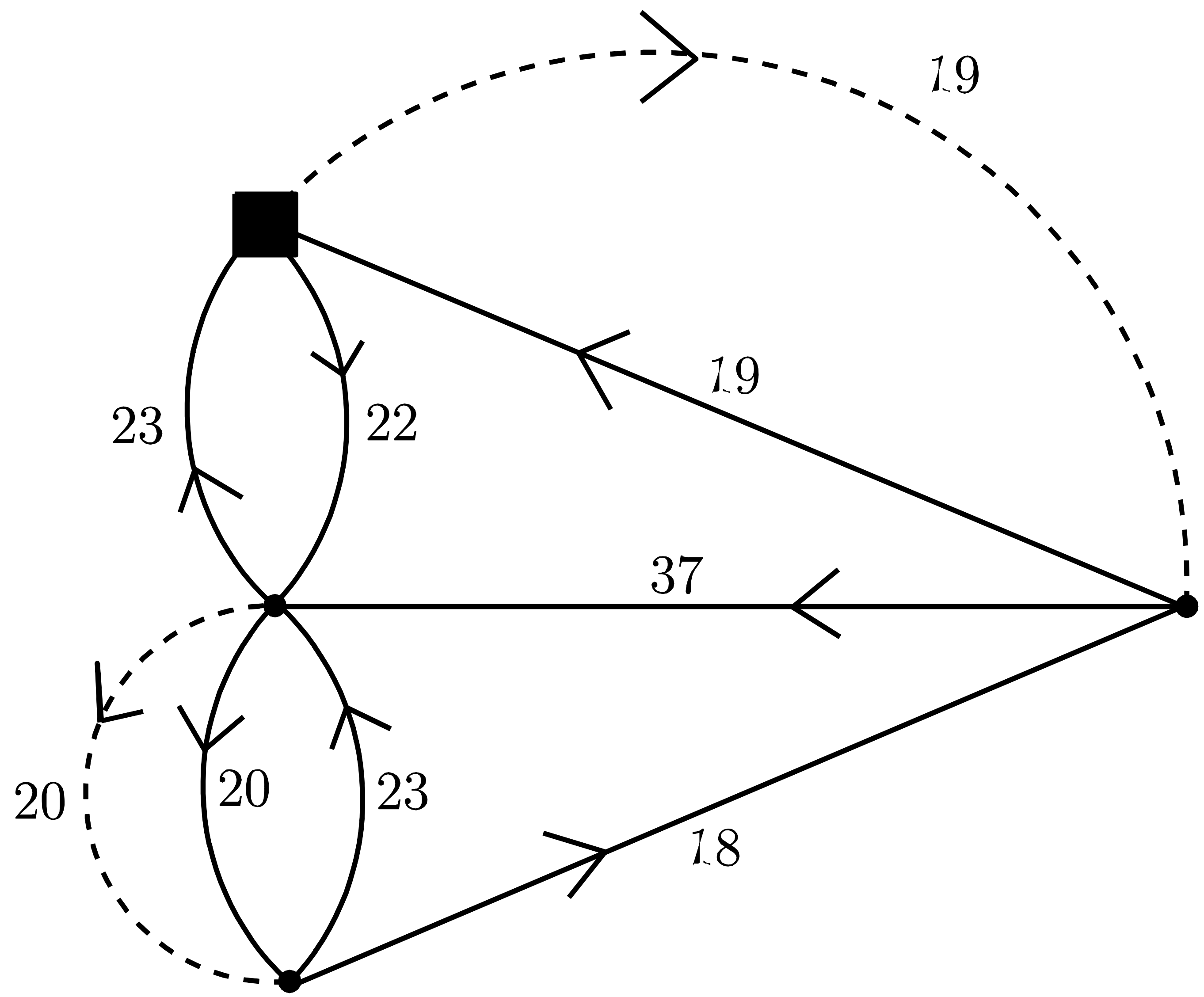
Task 1.4



Solutions

Task 1.5

There are many answers. Eg. 19, 19, 22, 20, 18, 37, 20, 23, 23



Solutions

Task 1.6

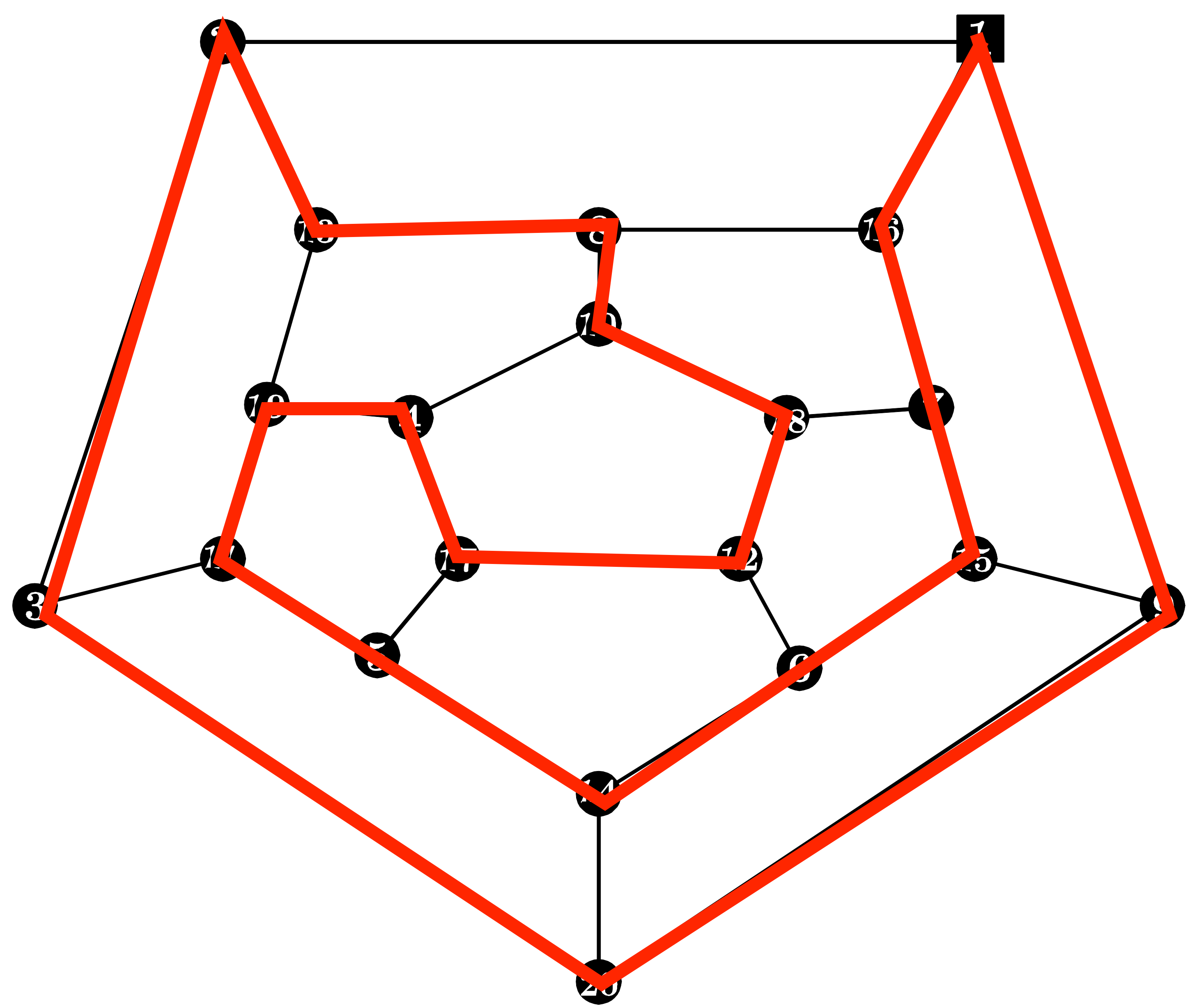
Suburb B has odd degree vertices. If no edge is repeated in a route then every vertex must have even degree.

This is because the route must enter AND leave every vertex.

Solutions

Task 2.1

There are many answers. Eg.



Solutions

Task 2.2

There are 5 different ways, all symmetrical, of choosing the edge that is not included. Suppose that edge (1,2) is excluded.

Then edges (2,13) and (1.16) must be in. Also all edges (19,11), (11,5), (5,14), (14,6), (6,15) and (15,7) must be in.

There are 2 possibilities:

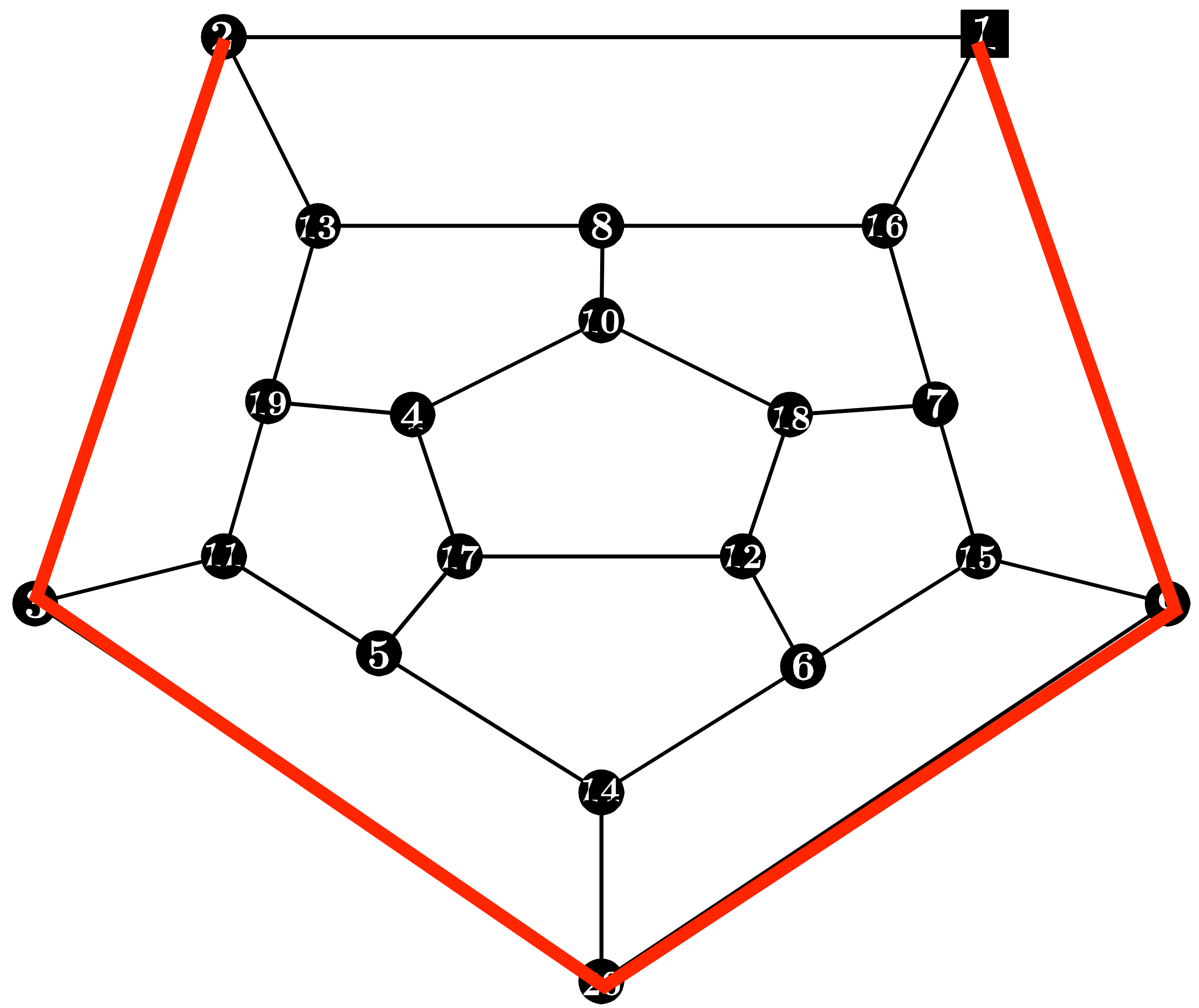
either edges (13,8) and (16,7) are in or edges (13,19) and (16,8) are in.

Both give unique Hamiltonian cycles from there.

This gives 5 x 2 = 10 cycles in total.

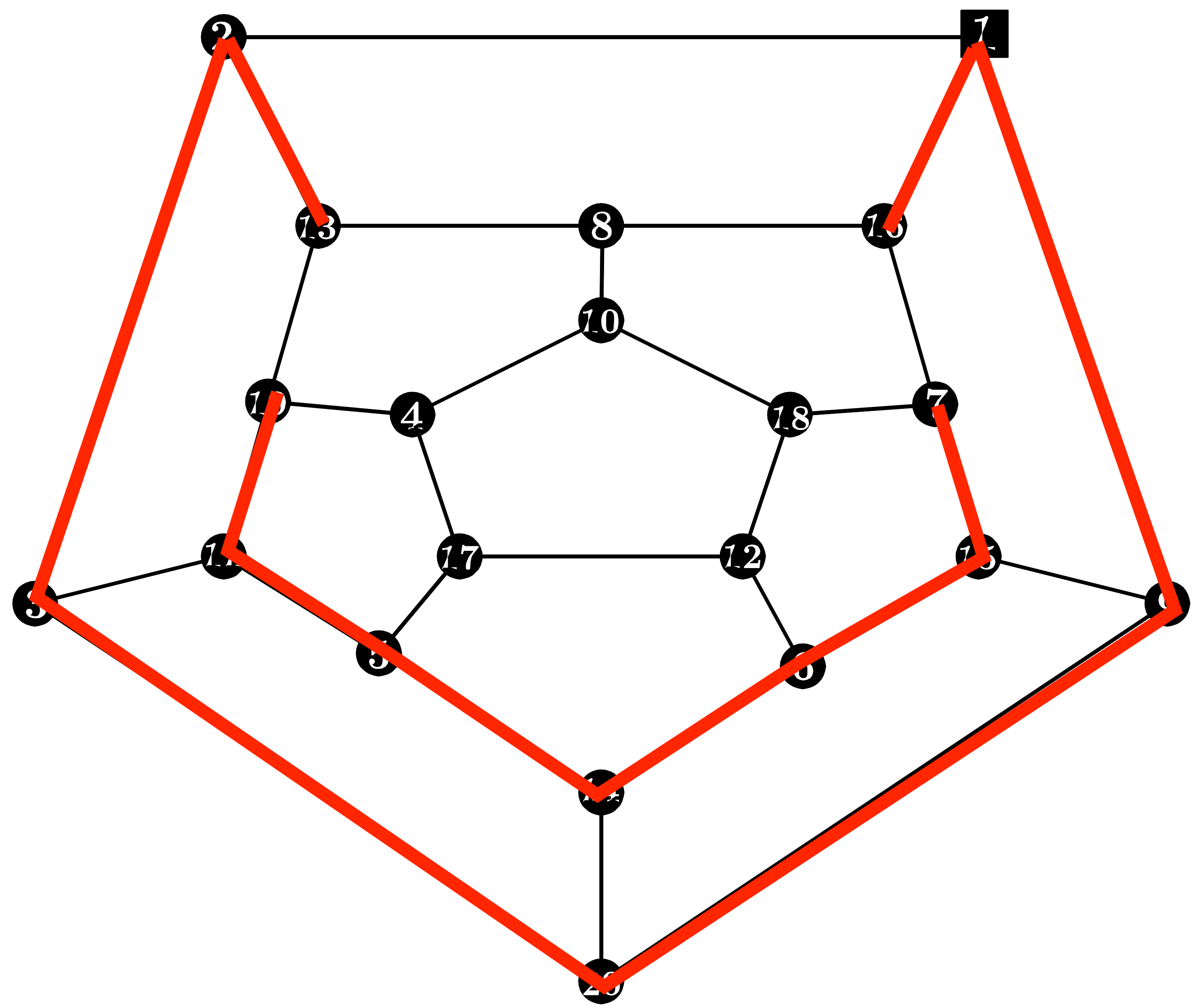
Solutions

Task 2.2



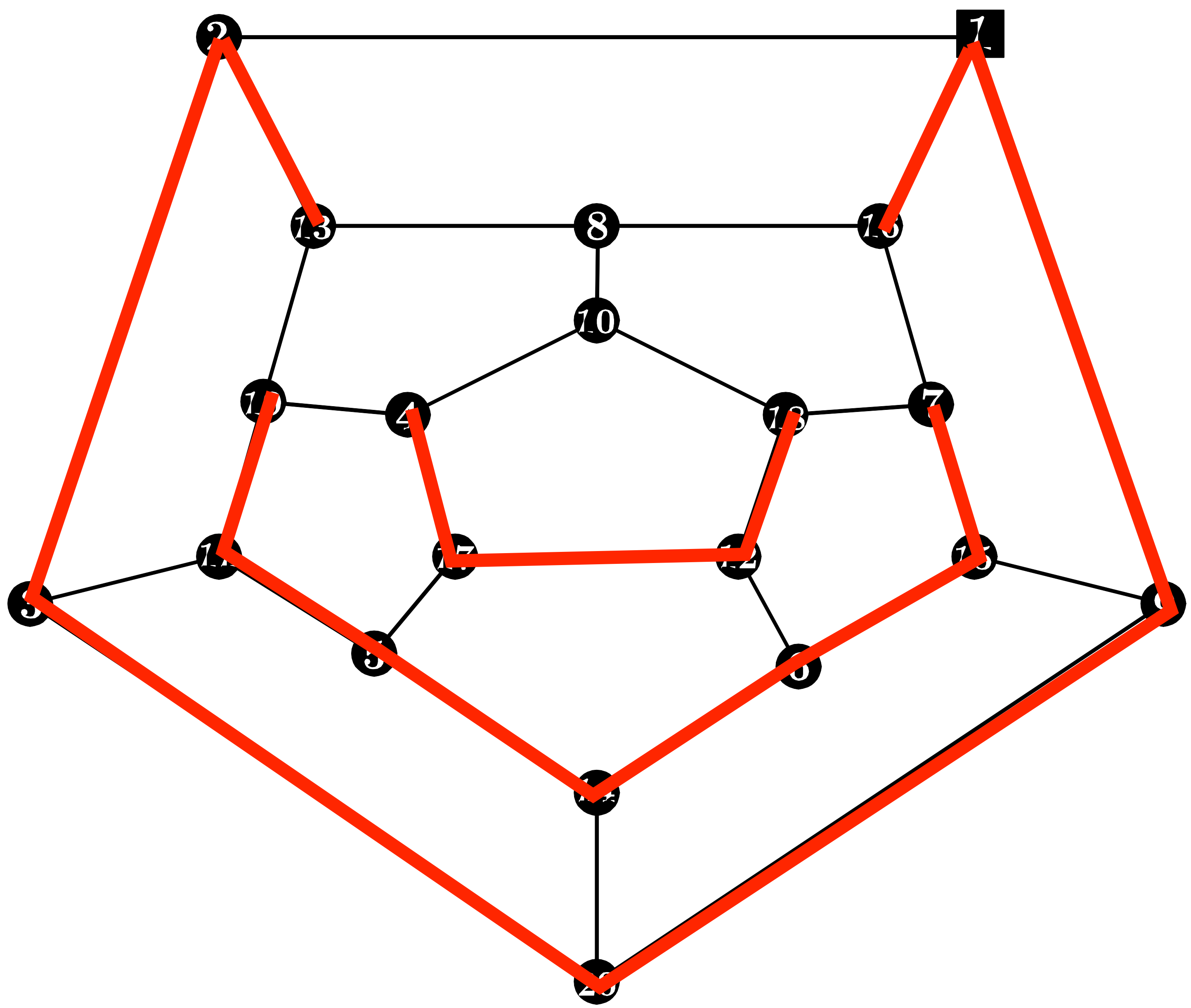
Solutions

Task 2.2



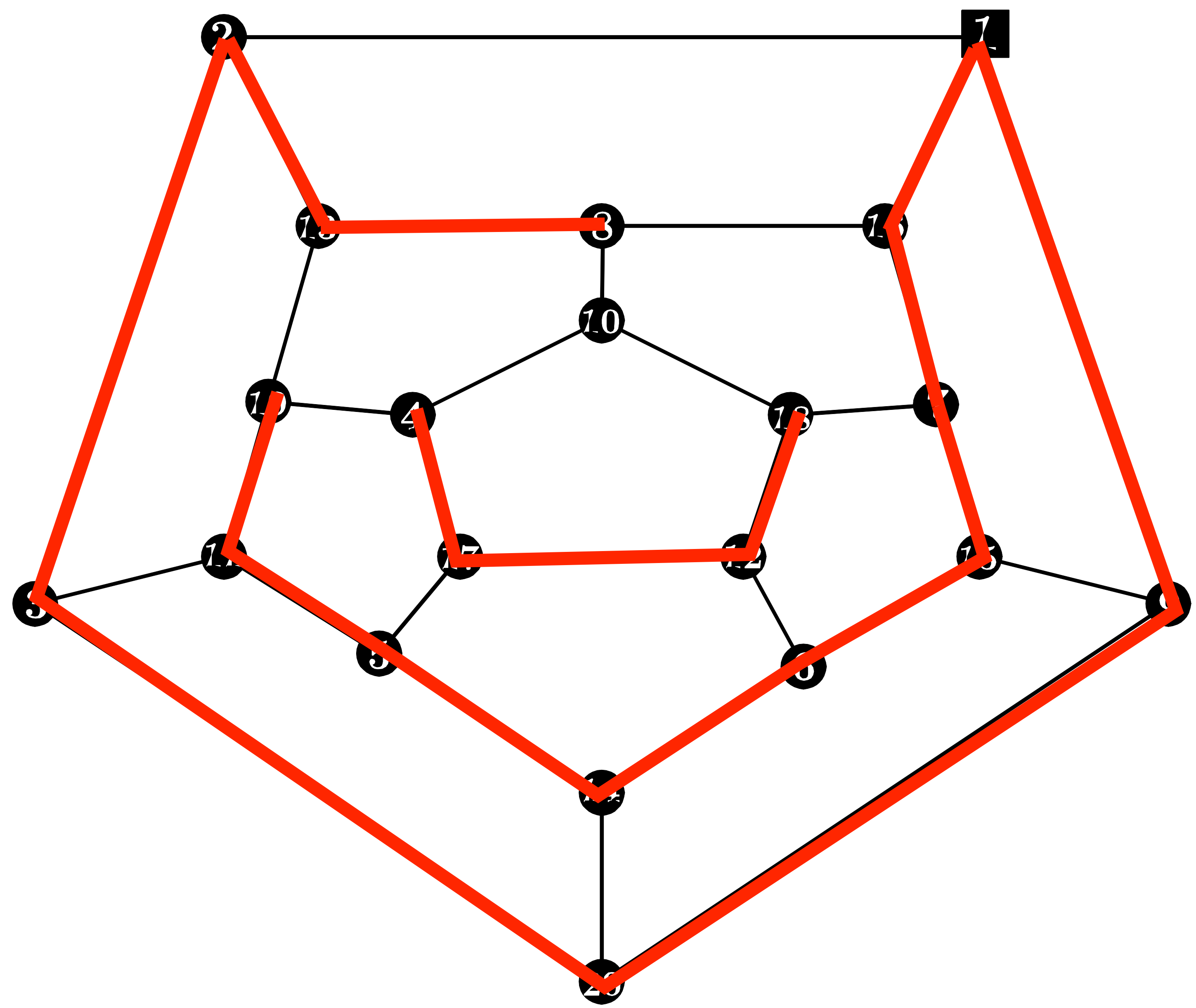
Solutions

Task 2.2



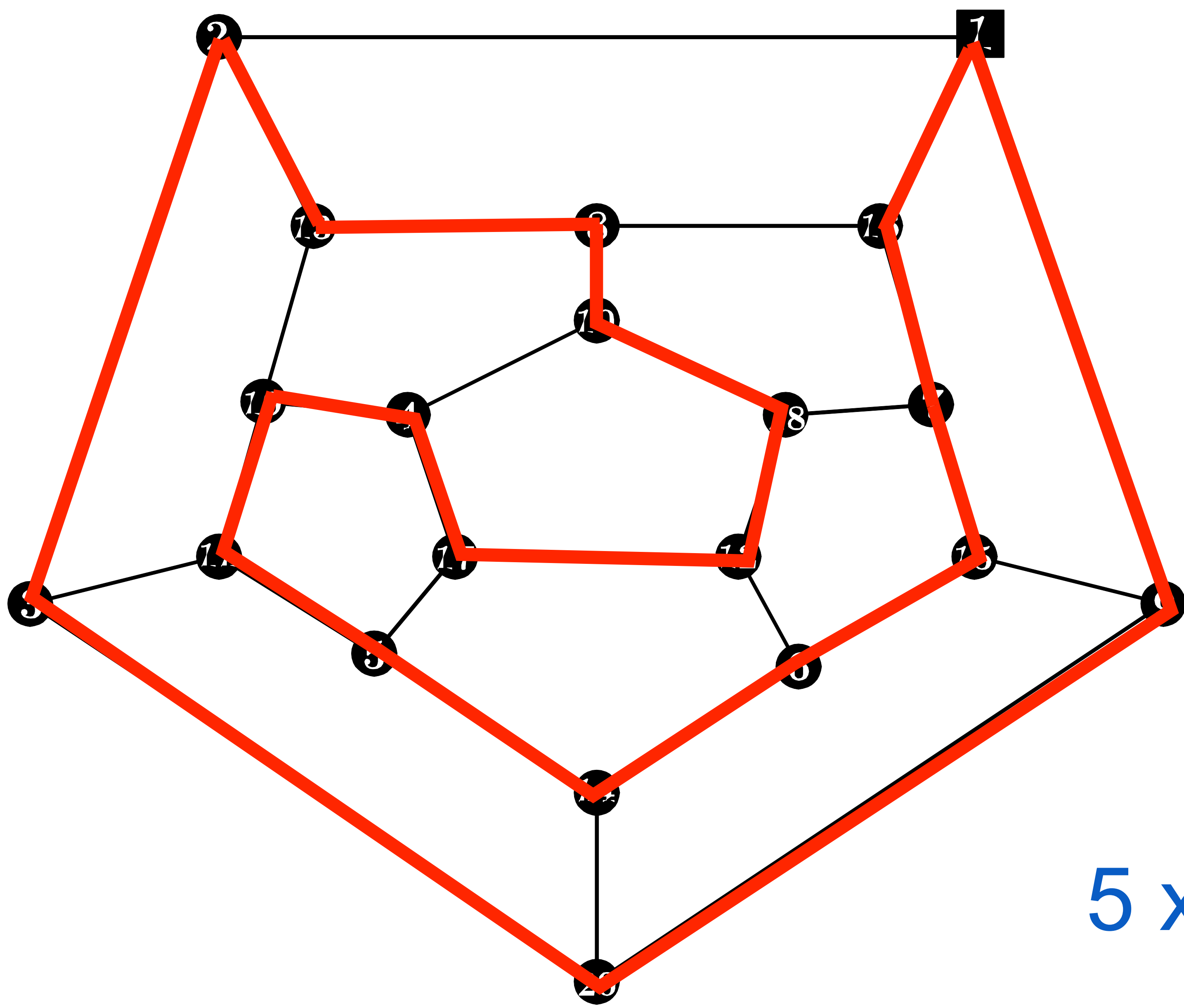
Solutions

Task 2.2



Solutions

Task 2.2



5 x 2 = 10

Solutions

Task 2.3

The graph is symmetrical. When we choose 4 edges on the outside boundary we get 4 edges on the inside one (and vice-versa).

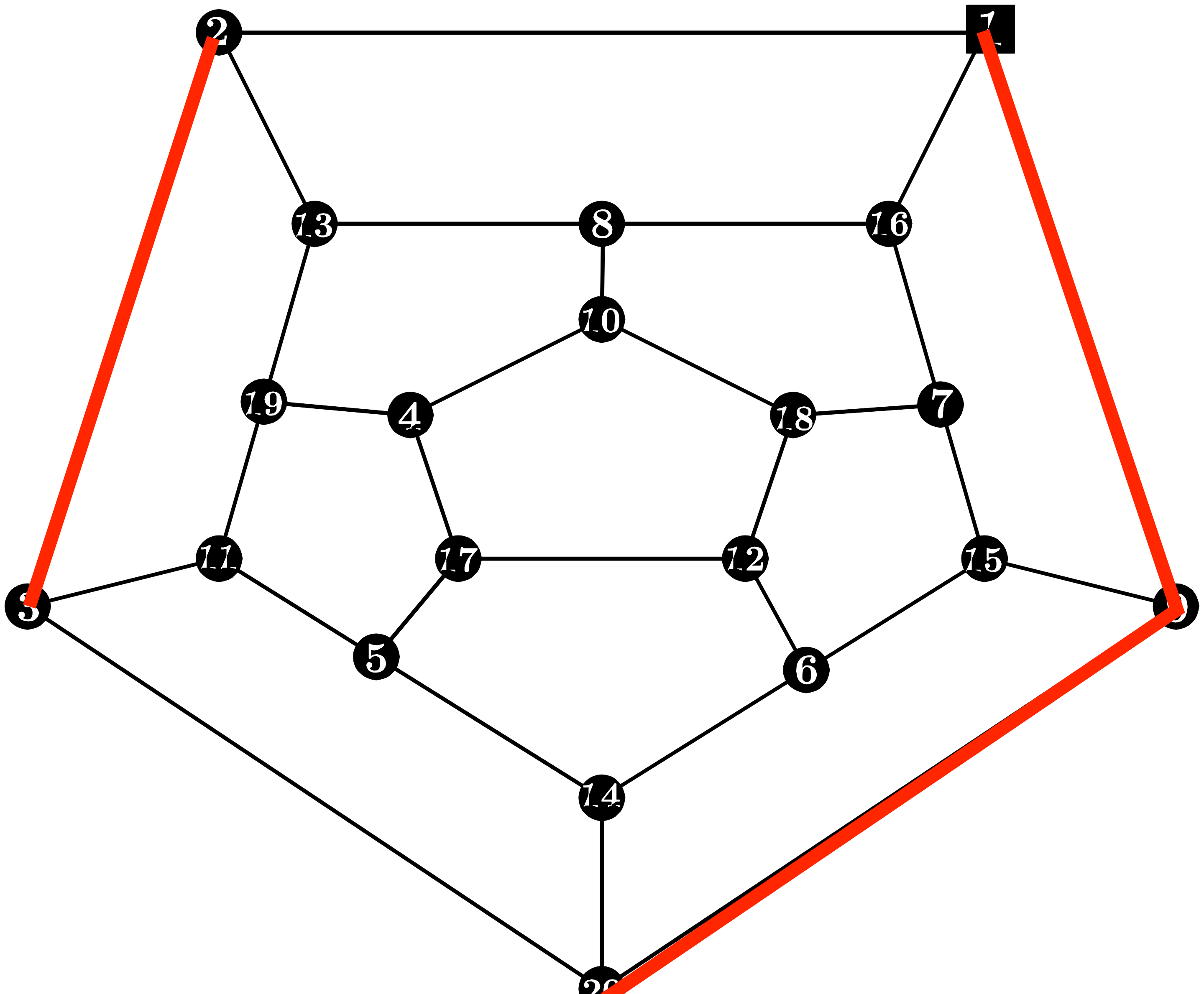
We cannot have more than 4 or fewer than 3 edges on a boundary. Therefore, we choose 3 edges on the outside boundary. There are 5 possibilities.

We then choose 3 edges on the inside pentagon. There are 5 possibilities but one of them is not valid. The other 4 give unique cycles.

So there are 5 x 4 = 20 cycles. This gives 20+10=30 cycles in total.

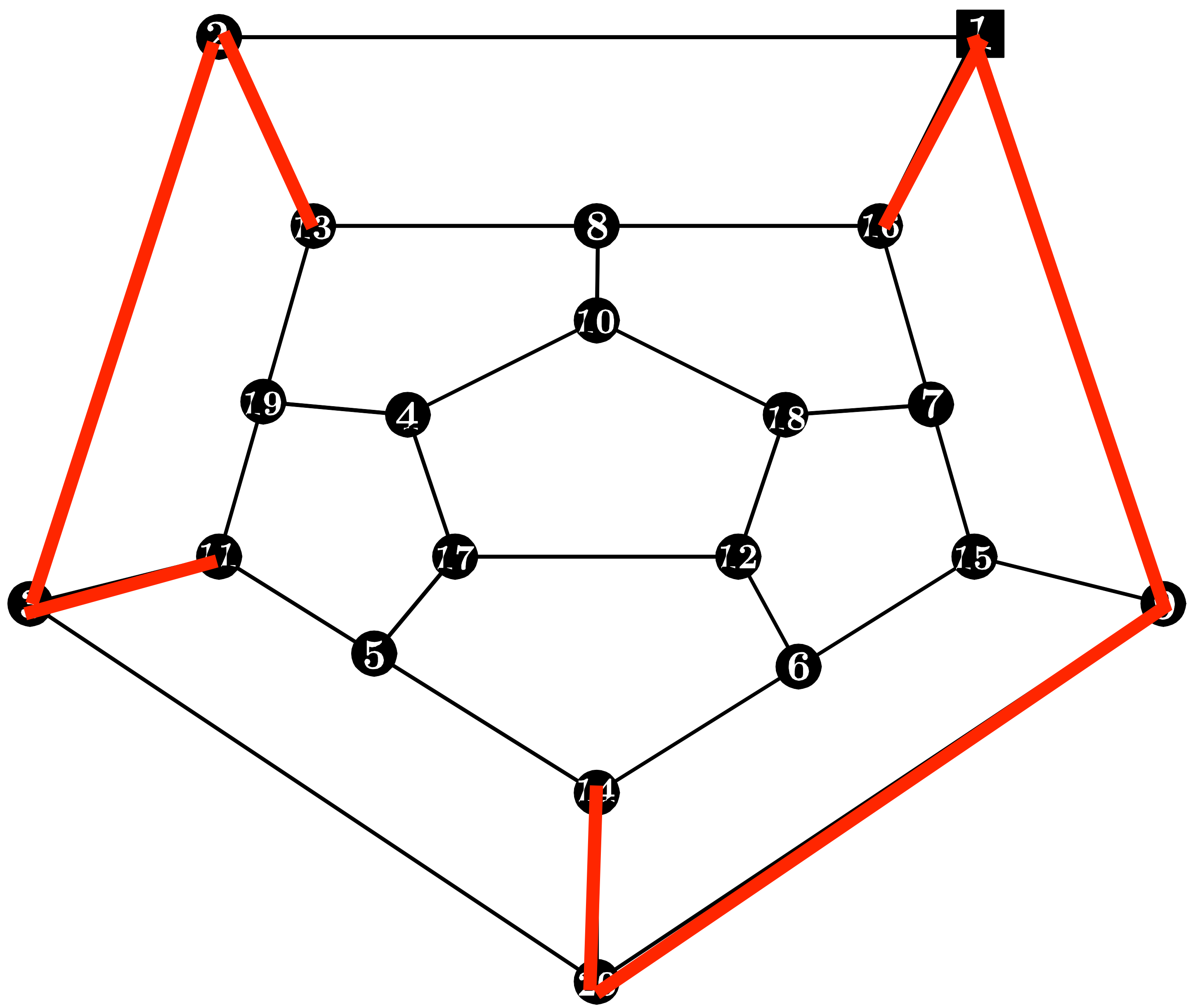
Solutions

Task 2.3



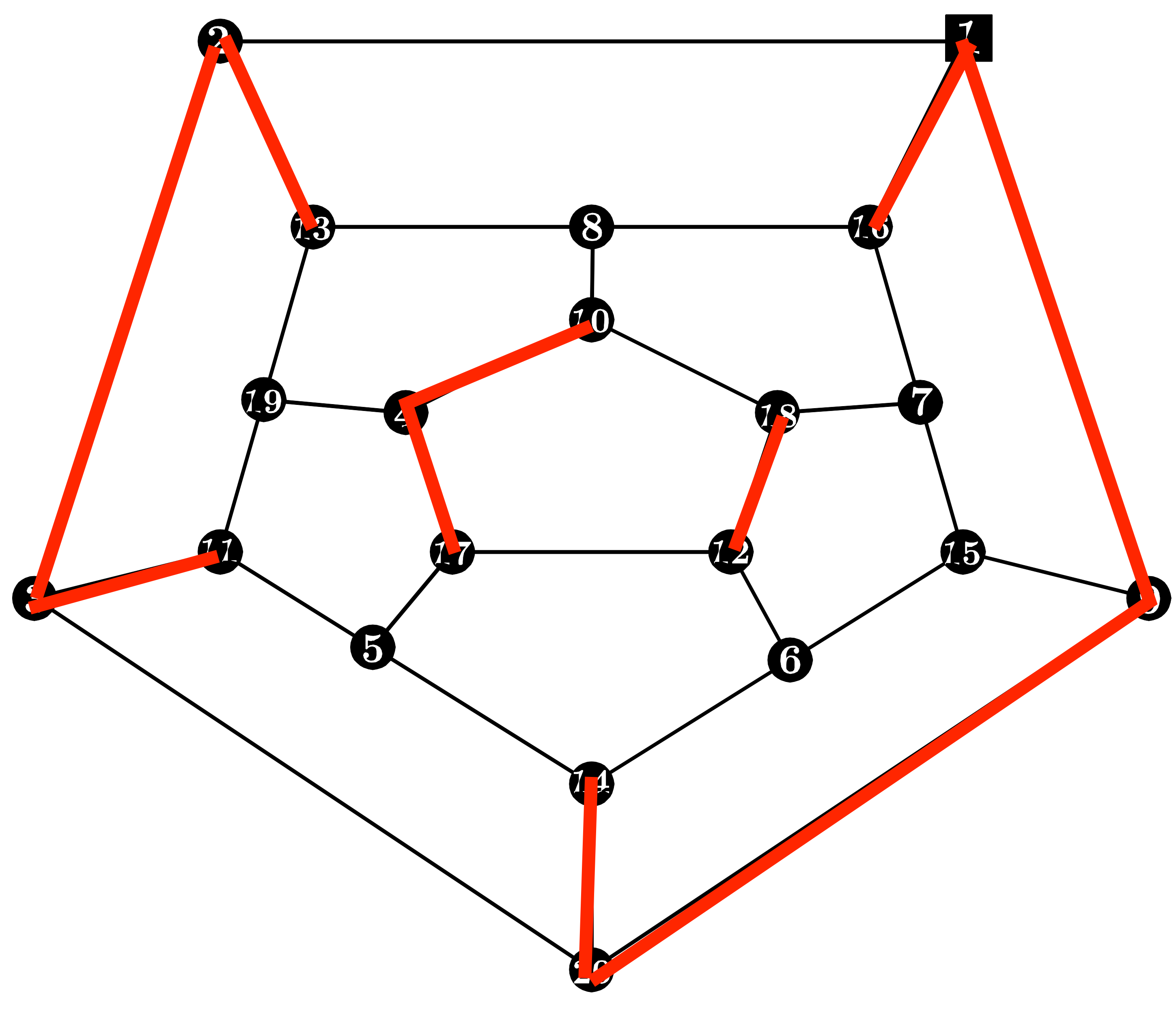
Solutions

Task 2.3



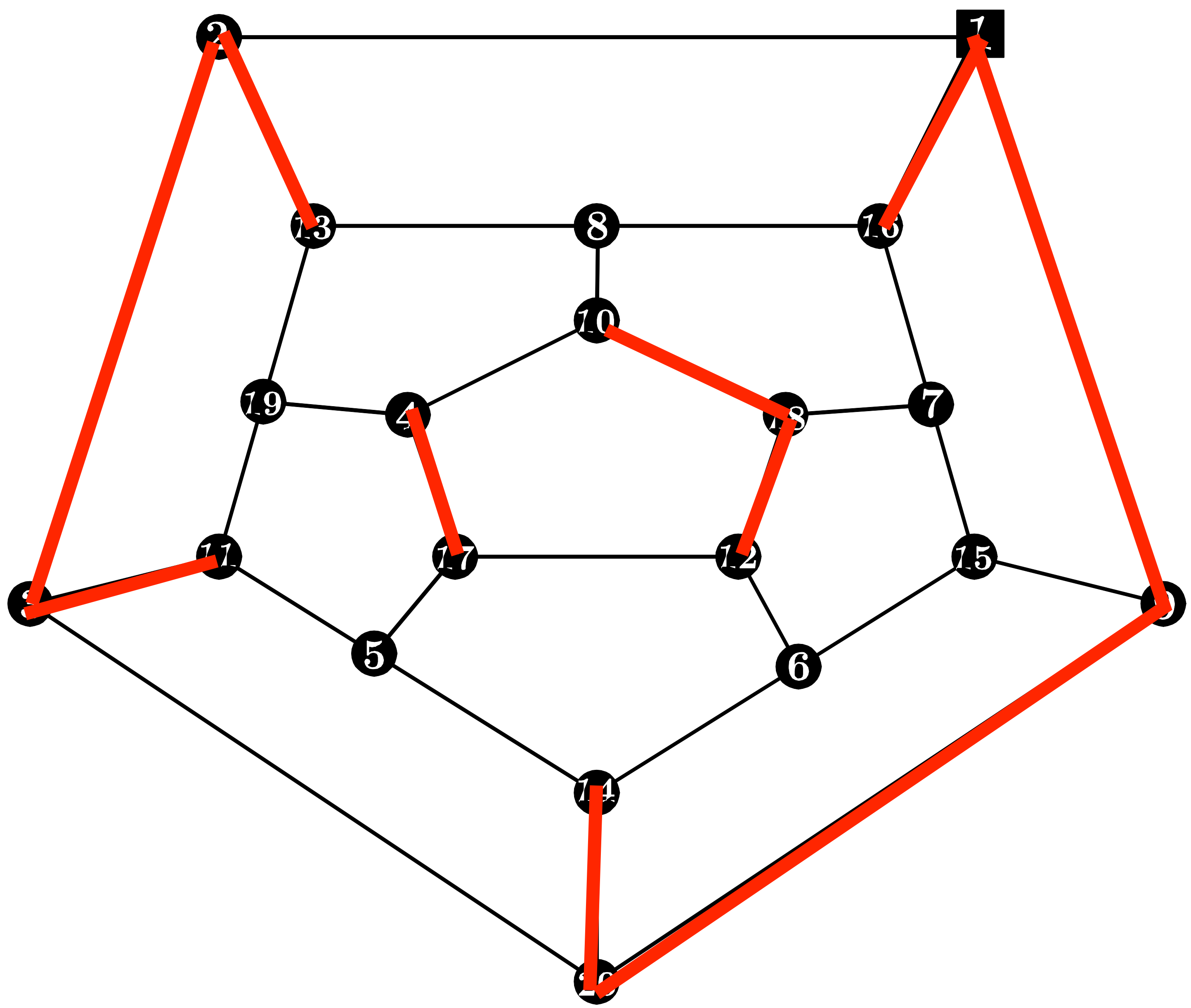
Solutions

Task 2.3



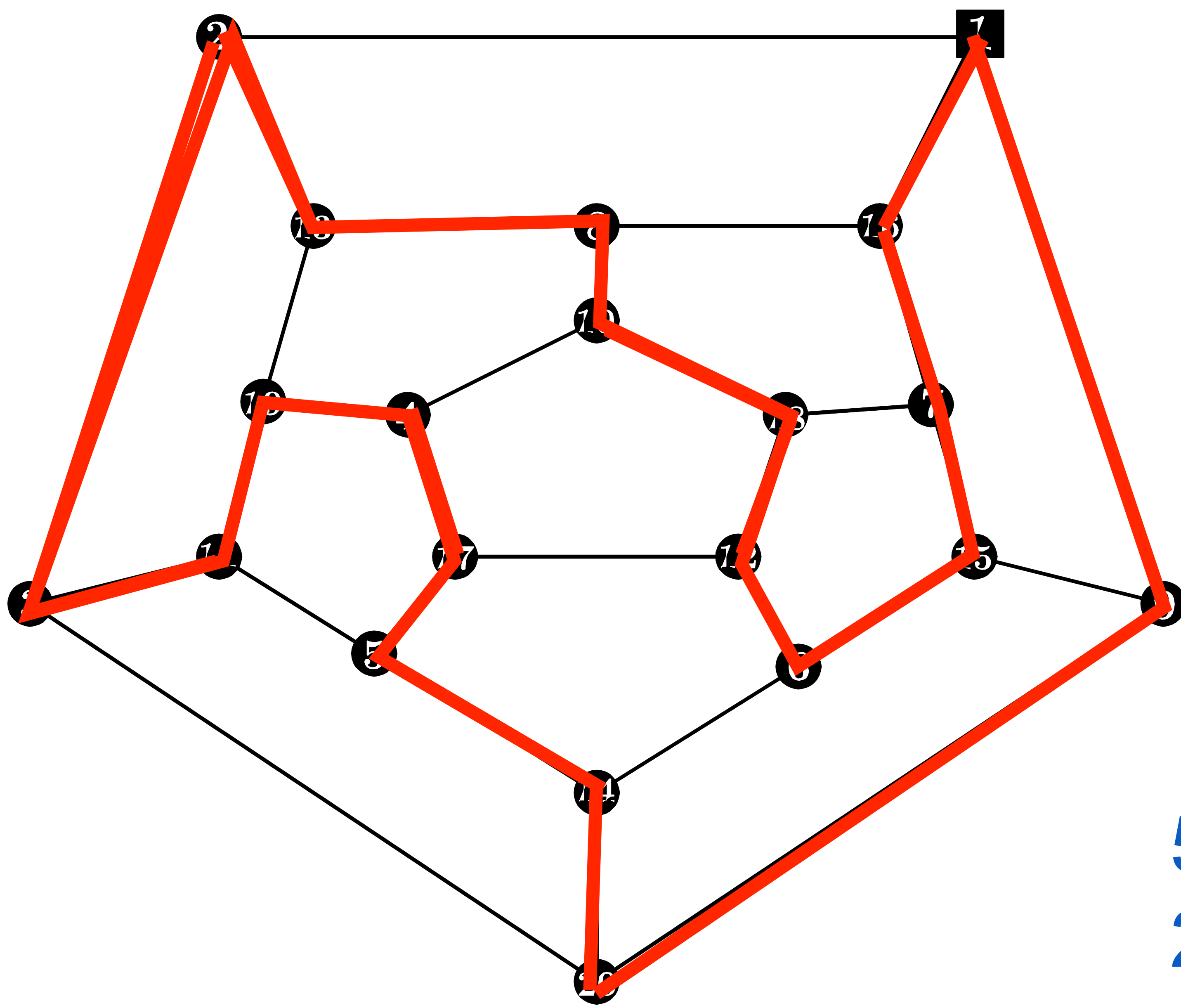
Solutions

Task 2.3



Solutions

Task 2.3



5 x 4 =20

20 + 10 =30