Mathematics in Industry and Technology (MIT) Challenge 2014: The postal distribution and delivery problems

Problem description

Australia Post delivers billions of letters each year to businesses and households. The delivery system consists of two main parts:

1. Local delivery of letters to postboxes.

2. Distribution of letters to post offices from a metropolitan distribution centre.

Both aspects of the delivery system must be optimised. The main running costs involved in the delivery system are for fuel and salaries. Both these costs are directly proportional to the total distance travelled by delivery vehicles. Australia Post can therefore achieve significant savings by choosing the delivery and distribution routes wisely. Your job will be to advise Australia Post on the best routes their delivery vehicles should take.

Task 1.
Let's first look at the local delivery aspect. To simplify the problem, we consider the route that a *single* delivery vehicle must take when delivering letters collected from the local post office. Consider Figure 1 which depicts the road network of a suburb, lets call it Suburb A, in Victoria. The local post office is represented by a black square. The lines represent streets, and each street has a number next to it representing the length of the street in hundreds of meters.

We will look at a few abstract properties of the problem first. Figure 1 is an example of what mathematicians refer to as a graph. Some more examples of graphs are shown in Figure 2. The small circles where lines meet are called *vertices* of the graph. The lines joining vertices are called *edges*. The number of edges meeting at a given vertex is referred to as the *degree* of the vertex.

For Task 1 you must now answer the following 6 questions.

1. What are the degrees of the vertices of the graph representing the network of local post offices in Figure 1? Provide you answer as concisely as possible.

2. In each of the graphs of Figure 2, how many odd-degree vertices are there? What do you notice? Draw a few more graphs and check that your observations hold for your graphs too.

3. Give a general argument for why your observation in Question 2 must be true.

4. If the delivery person is to deliver letters to households on all streets of Figure 1, what is the shortest possible route to achieve this? You may assume that the delivery person is able to carry all letters from the post office in a single load. Note that every street must be traversed at least once, and that the route must start and end at the post office. Provide your solution by listing the lengths of the streets in the order that they are traversed. Practice figures are provided at the end of this question paper.

5. As in the previous question, find an optimal route for the road network of Suburb B in Figure 3.

6. Observe that at least one road must be traversed more than once in Suburb B. What graph property makes this true of Suburb B but not of Suburb A? Provide an argument for why this is true.

Task 2.
Next we will look at the distribution component of the mail delivery system. To simplify the problem, we will consider the route that a single delivery truck takes as it delivers letter-bundles to post offices in a metropolitan region.

Figure 4 represents a hypothetical network of post offices connected by highways in Melbourne. For your convenience, the post offices have been labelled by numbers, but the order of the labels is not important. This time the task is to find the shortest route that visits each post office at least once. Not all roads (edges) need to be traversed, but every post-office (vertex) must occur on the route. The delivery truck must begin and end its journey at the mail distribution centre, which is the black square in Figure 4. For simplicity we assume that all roads have exactly the same length (note that the figure is NOT drawn to scale).

1. What route would you advise Australia Post to use for the network of post offices in Figure 4? Present your answer by highlighting the edges on the figure on the attached answer sheet. How many streets are traversed in your proposed route? Practice figures are provided at the end of the question paper.

2. You should have realised that there is more than one correct answer to the previous question. How many optimal routes are there that use exactly four edges on the boundary of the largest pentagon? Provide an argument for how you arrived at the answer. When referring to an edge use the notation (a, b), where a and b are the vertices at the end-points of the edge.

3. How many optimal routes are there in total? Once again, provide a concise yet rigorous argument for how you arrived at the answer. Hint: the graph is highly symmetrical.

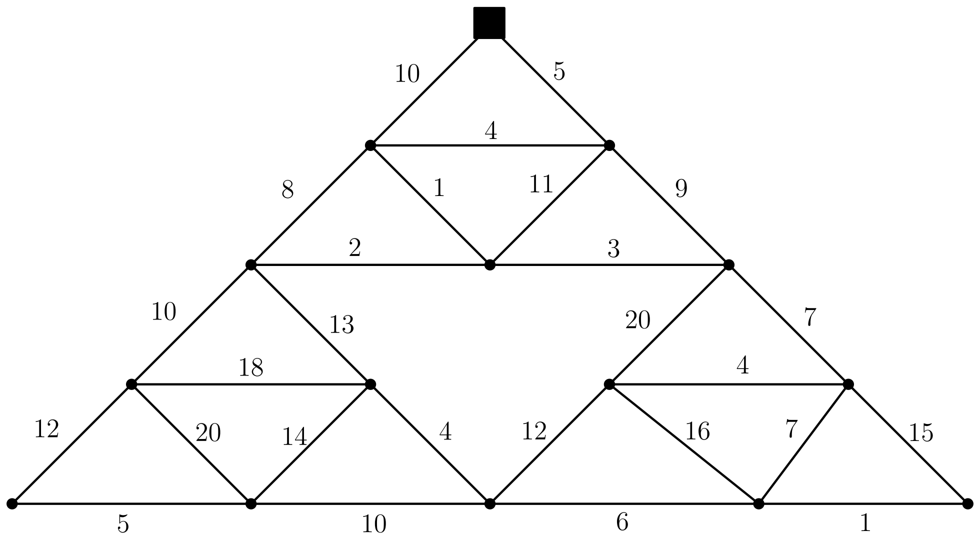

Figure 1: A graph representing the road network of Suburb A

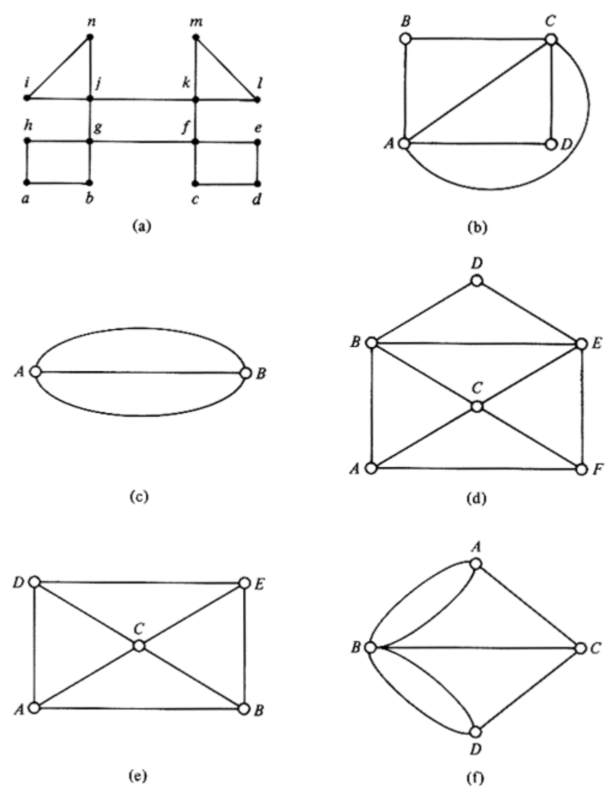

Figure 2: More examples of graphs

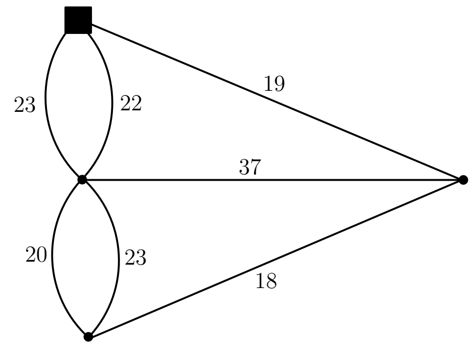

Figure 3: Graph representing the road network of Suburb B

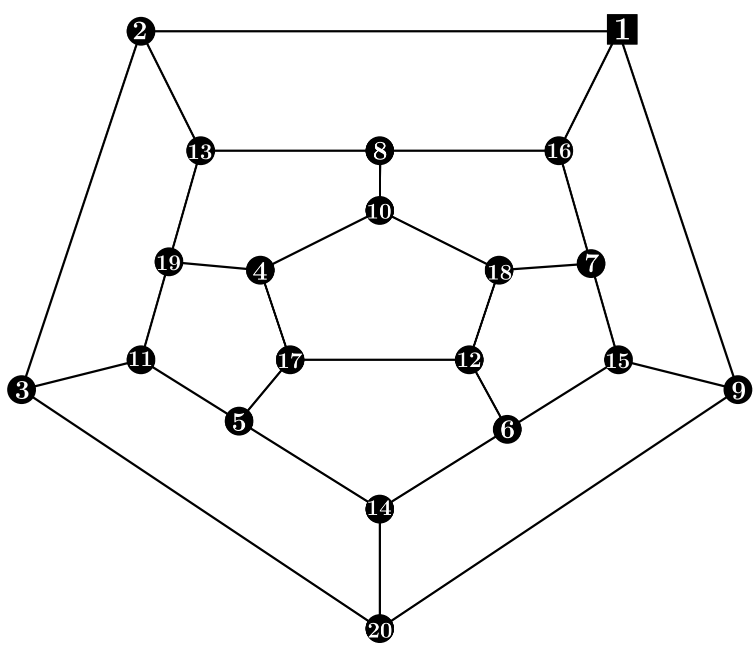

Figure 4: Graph representing the road network of all post-offices in metropolitan Melbourne

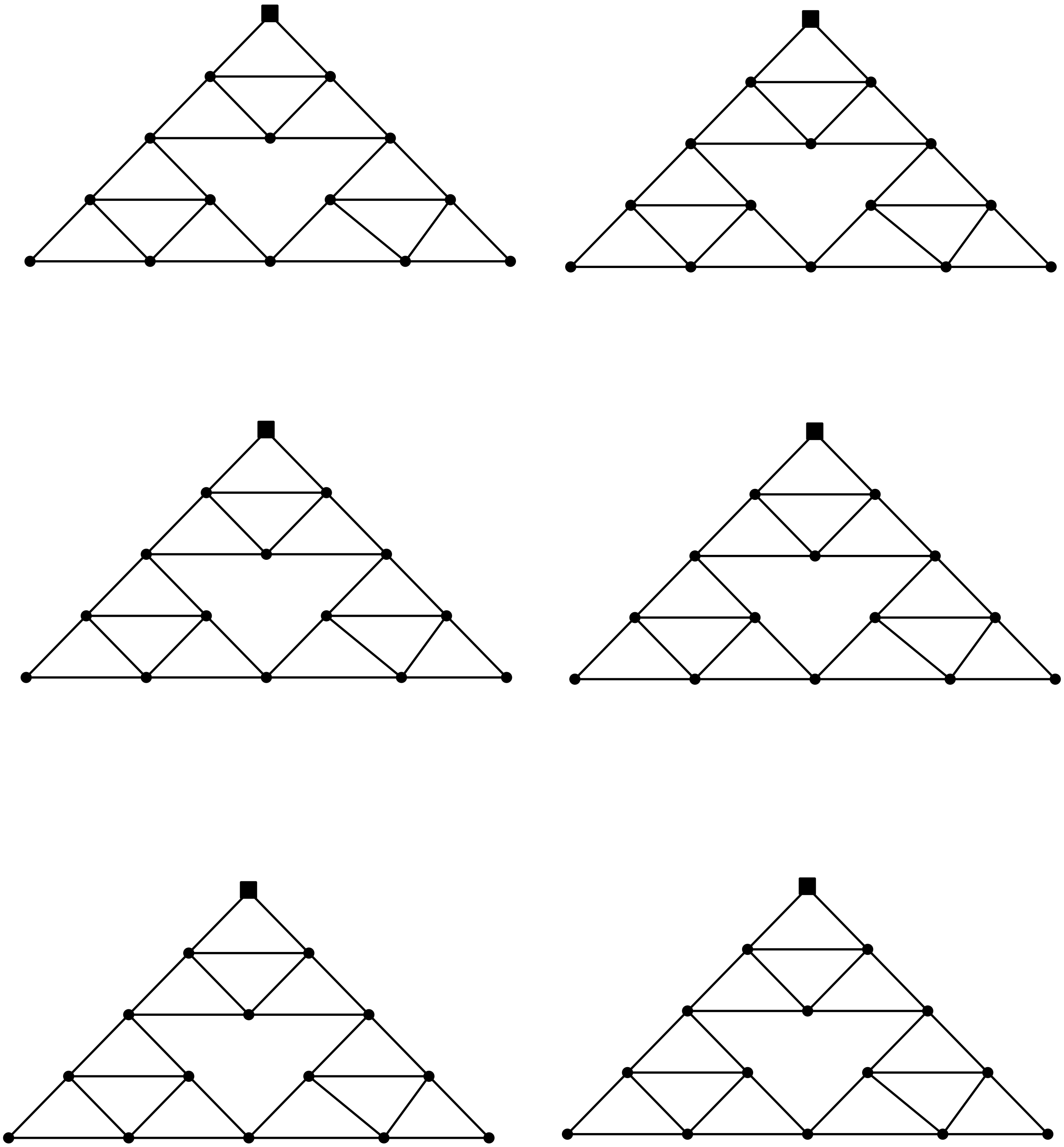

Figure 5: Practice figures

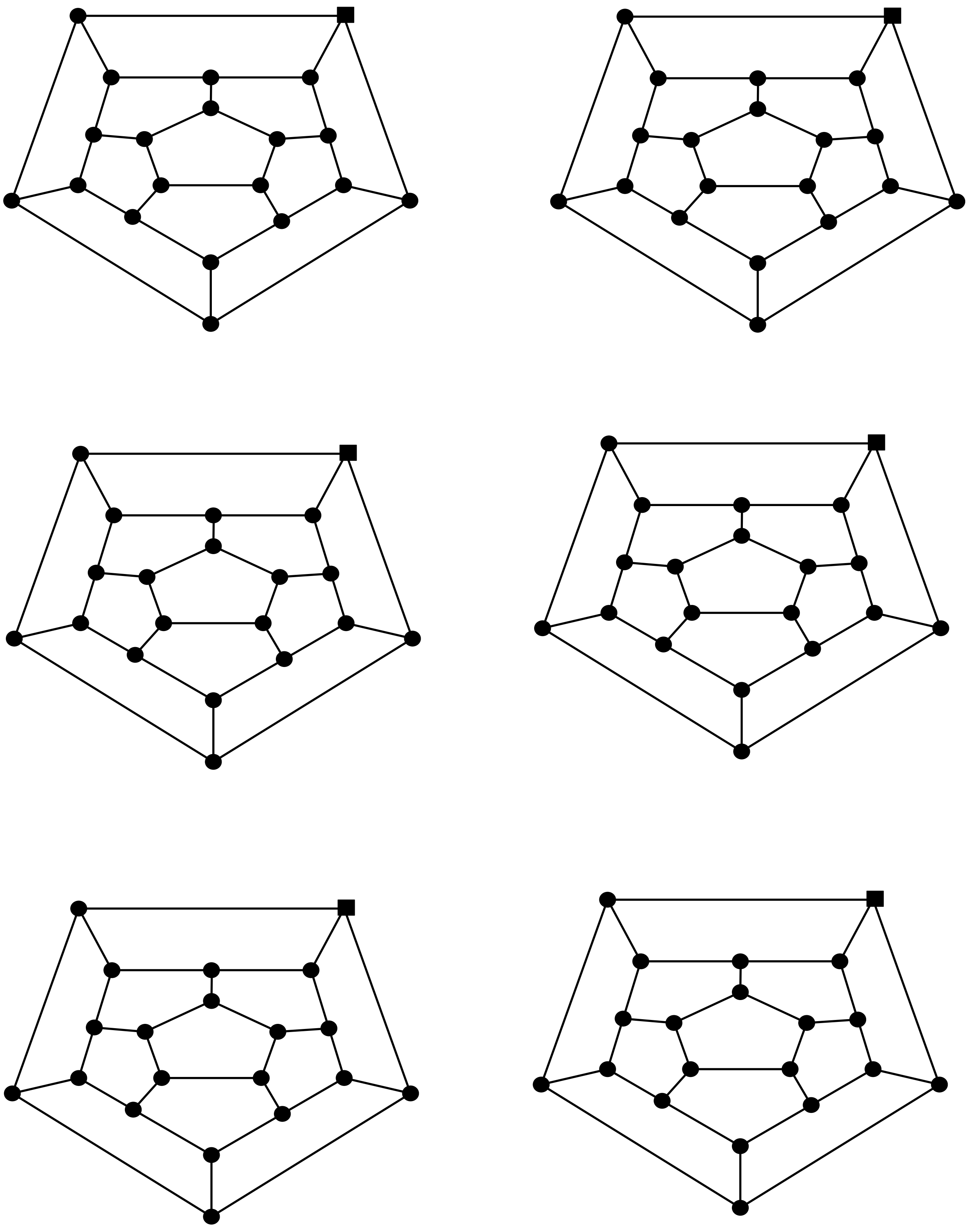

Figure 6: Practice figures